MATH 6A/D: HOMEWORK 26
GEOMETRY: RULER AND COMPASS CONSTRUCTIONS II
DEADLINE: FRIDAY, APRIL 30TH, 2021

CONSTRUCTIONS WITH RULER AND COMPASS

For our constructions we use two physical instruments: a ruler (to draw straight lines - random, through 1 point, through 2 points) and a compass (to draw circles of random radii, to draw circles of the predefined radius - like the distance between two points on our plane). Note that with these tools we don't measure anything against a calibrated scale of any sort, so no numbers please.

Also, we use two theoretical tools: congruence tests for triangles (SSS, SAS, ASA) and some properties of isosceles triangles (base angles are congruent, median, bisector and altitude is the same segment).

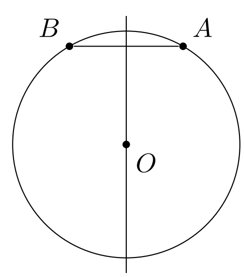
Here is a summary of operations we learned how to do using a ruler and compass. You can freely use any of them in the problems below.

1. Construct a perpendicular line to a line l on a plane
2. Given a line l and a point P outside of l, construct a perpendicular to l through P.
3. Given a line l and a point A on l, construct a perpendicular to l through A.
4. Construct the midpoint of a given segment AB.
5. Construct the perpendicular bisector of segment AB, i.e. a line that goes through the midpoint of AB and is perpendicular to AB.
6. Given an angle AOB, construct the angle bisector (i.e., a ray OM such that $\angle AOM \cong \angle BOM$). See the recipe and proof in our classroom Jamboard in Google Classroom.

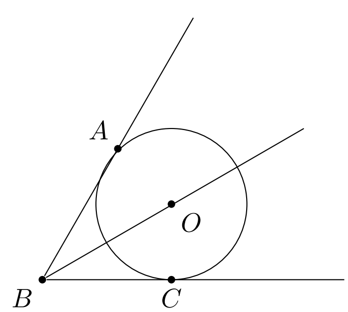
The following section explains the importance of these constructions.

PERPENDICULAR BISECTOR AND ANGLE BISECTOR

1. If two points A, B are on a circle, then the center of this circle lies on perpendicular bisector to AB (i.e., a line that goes through the midpoint of AB and is perpendicular to AB).



2. If a circle is inscribed in the angle ABC, then the center of this circle lies on the angle bisector.



HOMEWORK

When in the problems below, we say that some *length are given*, assume that there is an interval of a given length already drawn on the paper and you can set up your compass to draw circles of this radius. All constructions below are to be done using ruler and compass only! Notice that you are asked to explain why your recipe works. Please do not skip this part and do you best to explain it.

1. Given length a, construct an equilateral triangle with side a. Explain why your recipe works.

2. Given length a, construct a square with side a. Explain why your recipe works.

3. Given length a, construct a regular hexagon with side a. Explain why your recipe works.

4. Construct an isosceles triangle given base b and altitude h. Explain why your recipe works.

5. Construct the center of a circle given a circle O and two chords AB and CD. Explain why your recipe works.

[Hint: use the first statement from Perpendicular Bisector and Angle Bisector section above: If two points A, B are on a circle, then the center of this circle lies on perpendicular bisector to AB (i.e., a line that goes through the midpoint of AB and is perpendicular to AB).]

6. Construct a right triangle given a hypotenuse h and one of the catheti a. Explain why your recipe works.

7. (Geometry Lab) This lab weighs as much as three problems! Please read the description in the separate file called "Midsegment Toilet Paper". Follow the steps and take pictures. Submission of this lab means uploading no fewer than 3 photos AND sharing your takeaway from the process. Enjoy!

8. Six grasshoppers sit on a road. Every minute one grasshopper jumps 1 foot in one direction (along the road), and another grasshopper jumps 1 foot in the **opposite** direction. If initially the grasshoppers were at positions 1 ft, 2 ft, . . . , 6ft (measured from some point on the road), is it possible that after some time they all will all gather at the same place on the road?