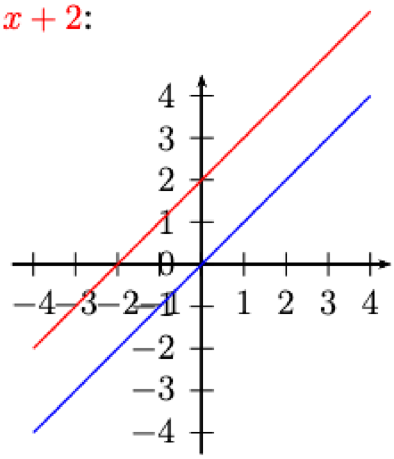
MATH 6A/D: HOMEWORK 25
DEADLINE IS FRIDAY, APRIL 23D, 2021

Graphs of functions

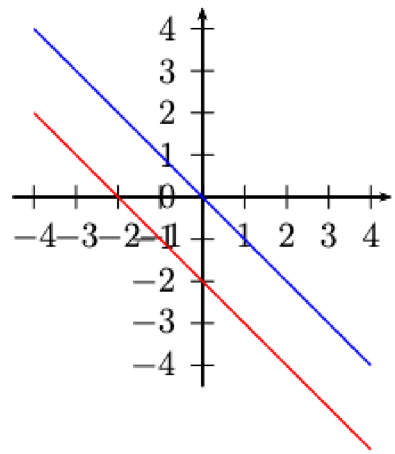
If the relation is of the form $y = f(x)$, where f is some function of x (i.e., some formula which contains x), the set of all points whose coordinates satisfy this relation is called the **graph** of f.

Line. Once again the graph of the function $y = mx + b$ is a straight line. The coefficient m is called the *slope*, b is the *y-intercept*.

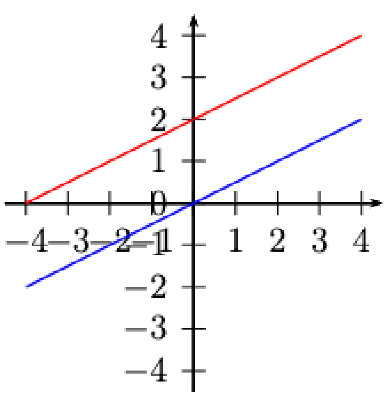
$y = x$; $y = x + 2$:



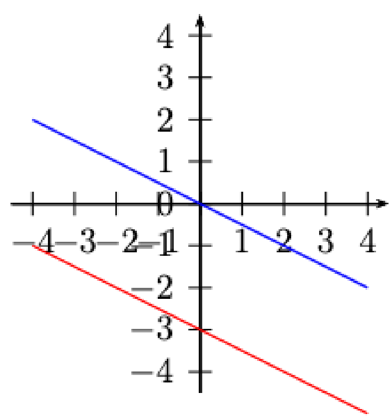
$y = -x$; $y = -x - 2$:



$y = \frac{1}{2}x$; $y = \frac{1}{2}x + 2$:

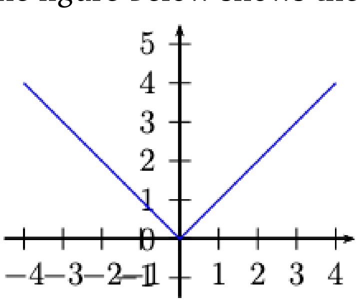


$y = -\frac{1}{2}x$; $y = -\frac{1}{2}x - 3$:



Graph of $y = |x|$

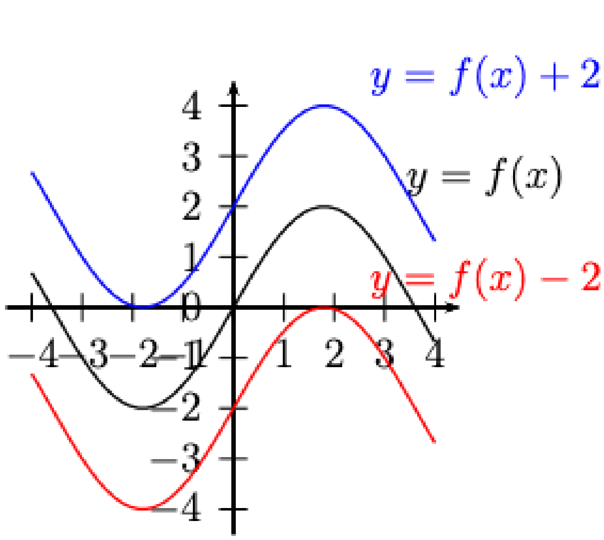
The figure below shows the graph of the function $y = |x|$ that we discussed in class.

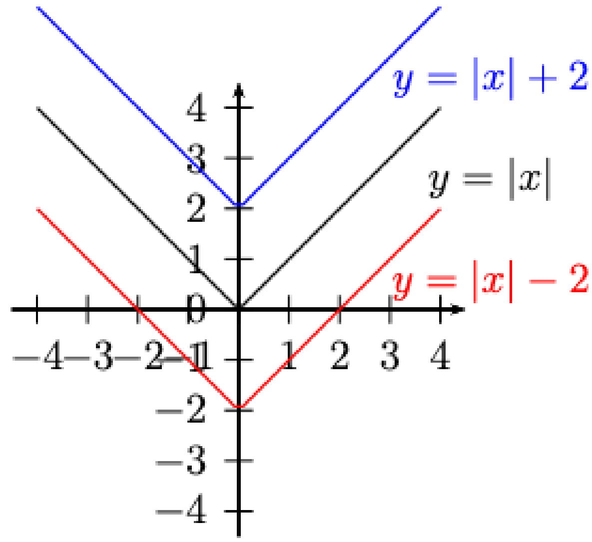


Trasnformations

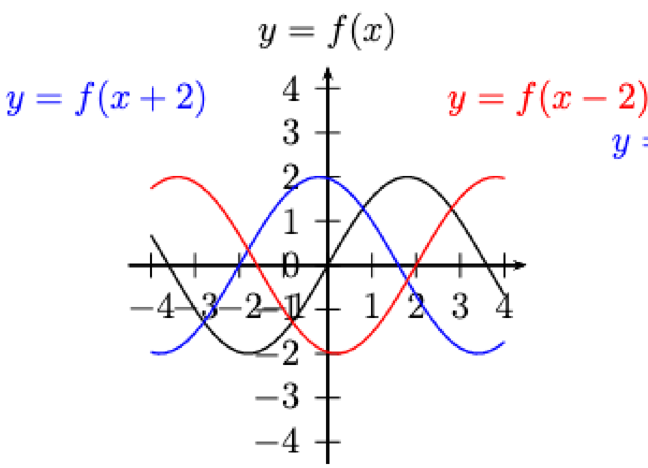
Having these basic graphs, we can produce new graphs, by doing certain transformations of the equations. Here are two of them.

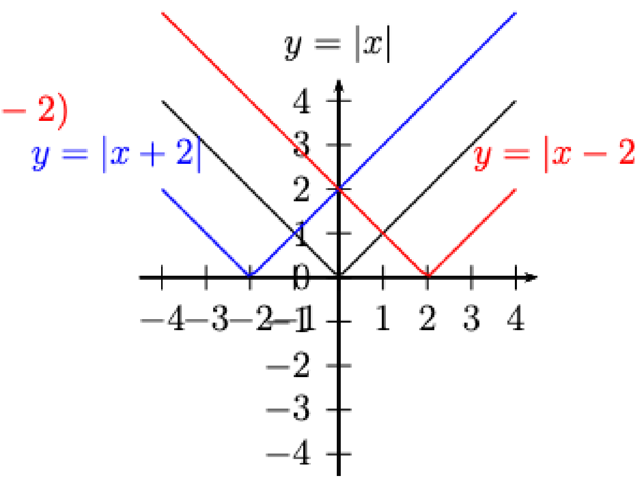
Vertical translations: Adding constant c to the right-hand side of equation shifts the graph by c units up (if c is positive; if c is negative, it shifts by $|c|$ down.)





Horizontal translations: Adding constant c to x shifts the graph by c units left if c is positive; if c is negative, it shifts by c right.





Constructions with ruler and compass

For the next couple of classes we will be mostly interested in doing the geometric constructions with ruler and compass. Note that the ruler can only be used for drawing straight lines through two points, not for measuring distances! We use the compass for drawing circles of various radii (but again we don't measure the radius against calibrated scale of any sort).

To solve a construction problem means:

- Give a recipe for constructing the required figure using only ruler and compass
- **Explain/confirm/prove why our recipe does give the correct answer**

For the first part, our recipe can use only the following operations:

- Draw a random line, a line through a point or through two given points
- Draw a circle of any radius with center at a random point or a given point
- Find and label on the figure intersection points of already constructed lines and circles.

For the second part, we will frequently use the results below.

Congruence tests for triangles

Recall that by definition, to check that two triangles are congruent, we need to check that corresponding angles are equal and corresponding sides are equal; thus, we need to check 6 equalities. However, it turns out that in fact, we can do with fewer checks.

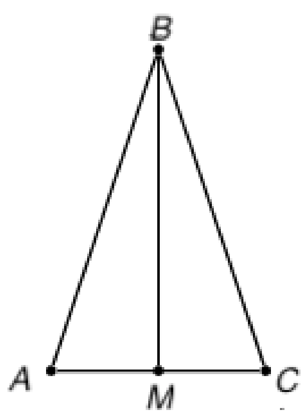
Axiom 1. (*SSS Rule*). If $AB = A'B', BC = B'C'$ and $AC = A'C'$ then $\triangle ABC \cong \triangle A'B'C'$.

Axiom 2. (*ASA Rule*). If $\angle A = \angle A', \angle B = \angle B'$ and $AB = A'B'$ then $\triangle ABC \cong \triangle A'B'C'$.

Axiom 3. (*SAS Rule*). If $AB = A'B', AC = A'C'$ and $\angle A = \angle A'$ then $\triangle ABC \cong \triangle A'B'C'$.

Isosceles Triangle

Recall that the triangle $\triangle ABC$ is called isosceles if $AB = BC$.



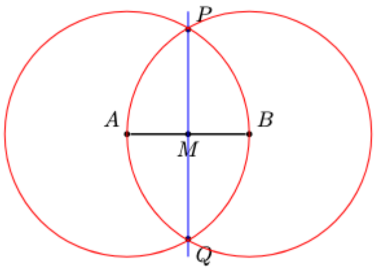
Theorem 1. Properties of an isosceles triangle:

1. In an isosceles triangle, base angles are equal: $\angle A = \angle C$.
2. In an isosceles triangle, let M be the midpoint of the base AC. Then line BM is also the bisector of angle B and the altitude: BM is perpendicular to AC.

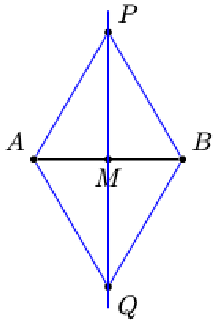
Example: Finding the midpoint of the line segment

Problem: given two points A, B, construct the midpoint M of the segment AB.
Construction:

1. Draw a circle with center at A and radius AB
2. Draw a circle with center at B and radius AB
3. Mark the two intersection points of these circles by P, Q
4. Draw line through points P, Q
5. Mark the intersection point of line PQ with line AB by M. This is the midpoint.



Analysis: This is a two-step argument. In this figure, triangles $\triangle APQ$ and $\triangle BPQ$ are congruent (*why?*), so the corresponding angles are equal:



From this, we can see that $\triangle APM \cong \triangle BPM$, so $AM = BM$.

Homework

1. (a) Sketch the graphs of functions $y = |x+1|$ and $y = -x + 0.25$.
(b) How many solutions do you think this equation has?

$$|x+1| = -x + 0.25$$

Note: you are not asked to find the solutions — just answer how many are there.

2. Sketch graphs of the following functions:

(a) $y = |x| + 1$ (b) $y = |x+1|$ (c) $y = |x-5| + 1$

3. Explain why in the construction above (the midpoint of a segment), triangles $\triangle APQ$ and $\triangle BPQ$ are congruent.

4. Explain why in the construction above (the midpoint of a segment), the line PQ will in fact be a perpendicular to AB.

5. Given a line l and a point P outside of l, construct a perpendicular to l through P.

6. Given a line l and a point A on l, construct a perpendicular to l through A.

7. (Geometry Lab) This lab weighs as much as three problems (3 points)! Please read the description in the separate file called "Radians Are Fun". Follow the steps and take pictures. Submission of this lab means uploading no fewer than 3 photos AND sharing your takeaway from the process. Enjoy!

***8.** Sketch the following functions:

(a) $y = |x| + |x+1|$ (b) $y = |x-1| + |x+1|$ (c) $|y| = x$

[Hint: Do draw graphs for (a) and (b), draw the graph of each of the summands, and then try to add the graphs