MATH 6A/D: HOMEWORK 21
DEADLINE: FRIDAY, MARCH 19TH, 2021

In our last class we talked about the Pi day, the significance of the number Pi and had a bit of fun with it. We didn't cover a lot of new material and didn't have time to discuss some of the homework problems that were difficult for many of you. Don't worry, we'll cover them in the next class. Meanwhile, please read carefully through this section to make sure you have all the basic information needed to solve the following homework problems.

DISTANCE BETWEEN POINTS

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the following formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This formula is a straightforward consequence of the Pythagorean Theorem.

MIDPOINT OF A SEGMENT

The **midpoint** M of a segment AB with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ has coordinates:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

GRAPHS OF FUNCTIONS

If the relation between y and x is in the form $y = f(x)$, where f is some function of x (i.e., some formula which contains x), the set of all points whose coordinates satisfy this relation is called the **graph** of f. We've worked with the graph of a linear equation/function a lot. Next time, we'll discuss the graph of $y = |x|$, some simple graph transformations and methods of solving systems of linear equations including the visual one I mentioned a couple of weeks ago.

CONVERTING BETWEEN SLOPE-INTERCEPT AND STANDARD FORM

If our linear equation is written in the form $y = mx + b$, where b is the y-intercept (i.e. the y-coordinate of the point where the line intersects the y-axis) and m is the slope of this line, this form is called the slope-intercept form of our linear equation.

If our linear equation is written in the form $ax + by = c$, where a, b and c are integers, don't have any common factors other than 1 and a is not negative, this form is called the standard form of our linear equation. Note that the listed requirements for the coefficients a, b and c are sometimes omitted.

We can easily convert the slope-intercept form to standard form and vice versa using the properties of equality.

For example, to convert the equation in the standard form $9x + 16y = 72$ to the slope-intercept form, we subtract $9x$ from both sides and divide both sides by 16. As a result, we get $y = \frac{9}{16}x + 4.5$, so the y-intercept is equal to 4.5 and the slope is equal to $\frac{9}{16}$.

To convert the equation in the slope-intercept form $y = \frac{2}{3}x + \frac{4}{7}$ to the standard form, we multiply both sides by 3 and by 7, subtract the x-term from both sides and multiply both sides by (-1) to make the a-coefficient not negative. As a result, we get $14x - 21y = -12$, where a, b and c are integers that do not have common factors other than 1 and a is not negative.

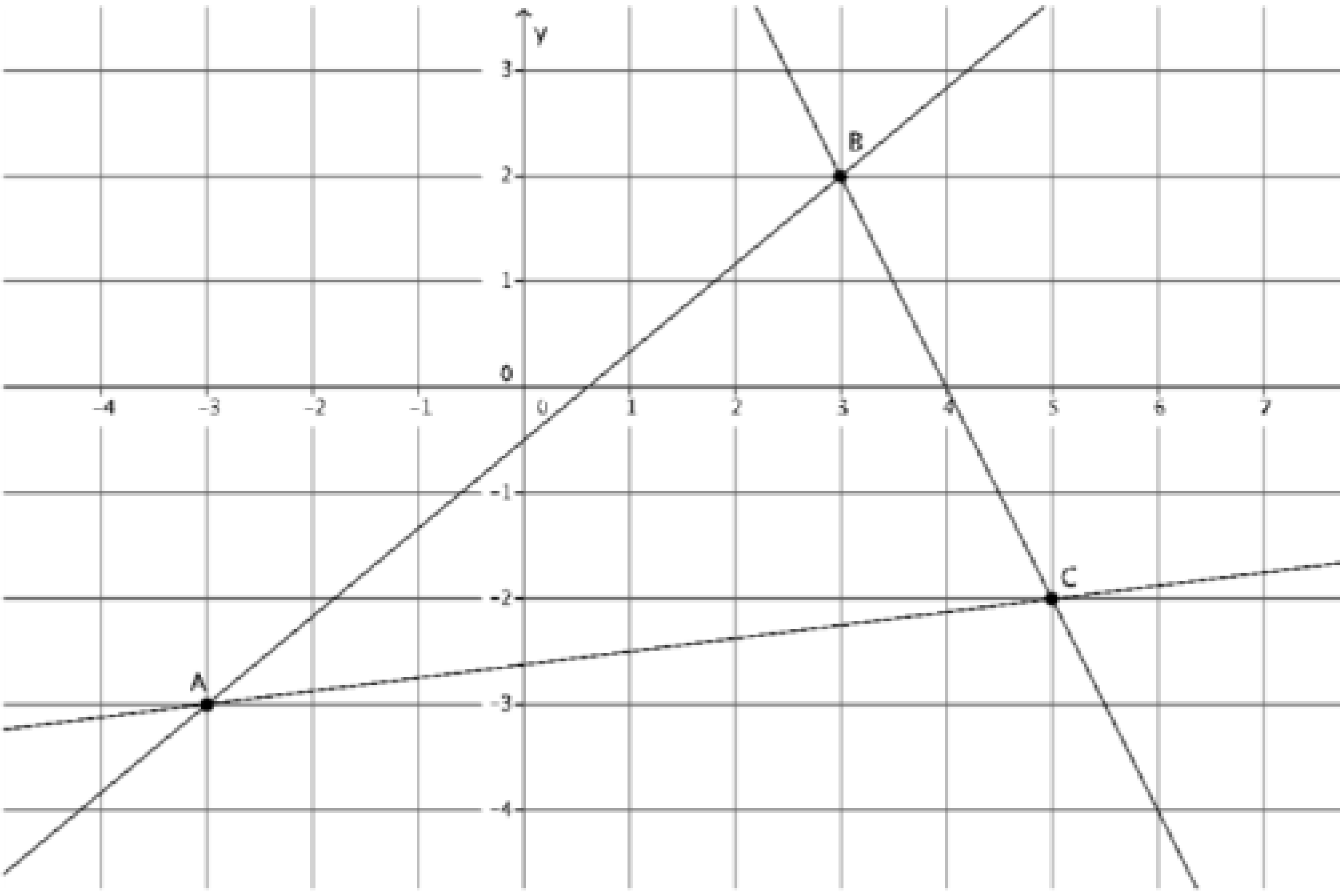
PROBLEMS

1. Convert from the slope-intercept form to standard form: $y = -\frac{1}{3}x - 9$. Sketch the graph.

2. Convert from the standard form to slope-intercept form: $5x + 2y = 20$. Sketch the graph.

3. Determine the equation of the line that goes through points $(4, -2)$ and $(-2, 4)$. Sketch the graph.

4. The triangle ΔABC is made up of the line segments formed from the intersection of lines L_{AB}, L_{BC} and L_{AC}. Write the equations that represent the lines that make up the triangle. Then, using the properties of equality change the equations from slope-intercept form $y = mx + b$ to standard form $ax + by = c$.



5. Let l_1 be the graph of $y = x+1$, l_2 be the graph of $y = x-1$, m_1 be the graph of $y = -x+1$, and m_2 be the graph of $y = -x-1$.

(a) Find the intersection point of l_1 and m_1; Label this point P and write down its coordinates.

(b) Find the intersection point of l_2 and m_2; Label this point P and write down its coordinates.

(c) Find the midpoint of AB and write down its coordinates.

(d) Let C be the intersection point of l_1 with m_2, and D be the intersection point of l_2 with m_1. What kind of quadrilateral is $ABCD$?

(e) Explain why l_1 and l_2 are parallel. What is the distance between them?

6. Prove using the Pythagorean theorem that $\overline{AC}$ is perpendicular to $\overline{AB}$ given points $A(-2,-2)$, $B(5,-2)$ and $C(-2,22)$. (Notation: $\overline{AB}$ is a segment from A to B, i.e. the part of a line from the point A to the point B.)

7. Consider the triangle ΔABC with the vertices $A(-2,-1)$, $B(2,0)$, $C(2,1)$. Find the coordinates of the midpoint of B and C. Find the length of the median (i.e. a median unites a vertex with the midpoint of the opposite side) from A in the triangle ΔABC.

8. Graph the equations $y = x-3$ and $y = -2x+6$ and find the coordinates of the intersection point of the lines.

9. Write the equation of the line through $(\sqrt{3}, \frac{5}{4})$ and

(a) Parallel to $y = 7$.
(b) Perpendicular to $y = 7$.
(c) Parallel to $\frac{1}{2}x - \frac{3}{4}y = 10$.
(d) Perpendicular to $\frac{1}{2}x - \frac{3}{4}y = 10$.

10. Consider the quadrilateral with vertices $(-2,-1), (2,2), (5,-2), (1,-5)$

(a) Show that the quadrilateral is a rectangle.
(b) Is the quadrilateral a square? Explain.
(c) What is the area of the quadrilateral?
(d) What is the area of the region of the quadrilateral that lies to the right of the y-axis?