**MATH 6A/D: HOMEWORK 20**
**DEADLINE: FRIDAY, MARCH 12TH, 2021**

## COORDINATE GEOMETRY: INTRODUCTION

In our last class we reviewed the basics of coordinate geometry. The basic notion is the **coordinate plane** – a plane with a given fixed point, called the **origin**, as well as two perpendicular lines – **axes**, called the $x$**-axis** and the $y$**-axis**. $x$-axis is usually drawn horizontally, and $y$-axis — vertically. These two axes have a **scale** – "distance" from the origin.

The scales on the axes allow us to describe any point on the plane by its **coordinates**. To find coordinates of a point $P$, draw lines through $P$ perpendicular to the $x$- and $y$-axes. These lines intersect the axes in points with coordinates $x_0$ and $y_0$. Then the point $P$ has $x$-coordinate $x_0$, and $y$-coordinate $y_0$, and the notation for that is: $P(x_0, y_0)$. You can find many examples of plotted points in our class Jamboard.

For this homework I strongly recommend using **graph paper** since you'll be asked to plot points and draw graphs of linear equations (lines) on the coordinate plane. Using graph paper is the most accurate way to do it by hand. I know that some of you are familiar with some online graphic calculators and other graphing tools. Please do not use them so far, you'll have plenty of time to do it in the future. In this homework I really want you to show me that you can do it by hand. Thank you!

## LINEAR EQUATIONS ON THE COORDINATE PLANE

Given some relation which involves variables $x$, $y$ (such as $x + 2y = 0$ or $y = x^2 + 1$), we can plot on the coordinate plane all points $M(x, y)$ whose coordinates satisfy this equation. Of course, there will be infinitely many such points; however, they usually fill some smooth line or curve. This curve is called the **graph** of the given relation.

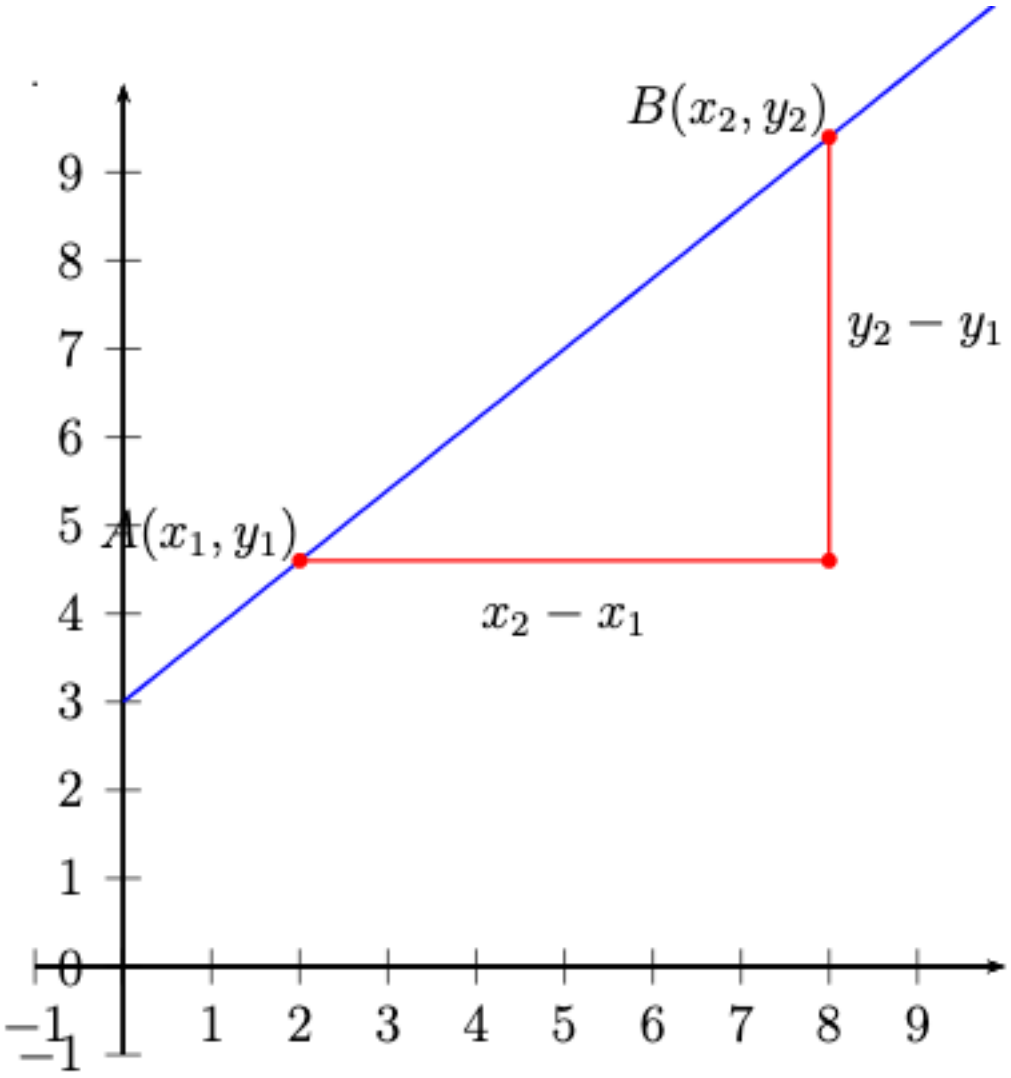
Every relation (**equation**) of the form:

$$y = mx + b$$

where $m$, $b$ are some numbers, defines a straight line. Notice that it is just one example of a linear equation, there are many ways to write the same x,y-relation as a linear equation (see the class Jamboard). If our linear equation is written in the form $y = mx + b$, $b$, the slope of this line is determined by $m$. As you move along the line, $y$ changes $m$ times as fast as $x$, so if you increase $x$ by $1$, then $y$ will increase by $m$:

In other words, given two points $A(x_1, y_1)$ and $B(x_2, y_2)$ **slope** can be computed by dividing change of $y$: $y_2 - y_1$ by the change of $x$: $x_2 - x_1$.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$




Two non vertical lines are **parallel** if and only if they have the **same slope**.

In the equation $y = mx + b$, $b$ is a $y$**-intercept**, and determines where the line intersects the vertical axis ($y$-axis). That's why the form $y = mx + b$, $b$ is called the **slope-intercept form**.

The equation of the **vertical** line is $x = k$, and the equation of the **horizontal** line is $y = k$. Notice that in case of the vertical line, the slope is undefined.

## PROBLEMS

1. A point $B$ is 5 units above and 2 units to the left of point $A(7, 5)$. What are the coordinates of point $B$?
2. What is the slope of a line whose equation is $y = 2x$? What is the slope of a line whose equation is $y = mx$?
3. Find the equation of the line through $(1, 1)$ with slope $2$.
4. Find the equation of the line through points $(1, 1)$ and $(3, 7)$. [Hint: what is the slope?]
5. In this problem you will find equations that describe some lines.
   (a) What is the equation whose graph is the $y$-axis?
   (b) What is the equation of a line whose points all lie 5 units above the $x$-axis?
   (c) Is the graph of $y = x$ a line? Draw it.
   (d) Find the equation of a line that contains the points $(1, -1)$, $(2, -2)$, and $(3, -3)$.

**6.** (a) Find $k$ if $(1, 9)$ is on the graph of $y - 2x = k$. Sketch the graph.
(b) Find $k$ if $(1, k)$ is on the graph of $5x + 4y - 1 = 0$. Sketch the graph.

**7.** Draw points $A(4, 1)$, $B(3, 5)$, $C(-1, 4)$. If you did everything correctly, you will get 3 vertices of a square. What are coordinates of the fourth vertex? What is the area of this square? [Hint: use the Pythagorean theorem to calculate the length of the side of the square.]

**8.** (a) 3 points $A(0, 0)$, $B(1, 3)$, $D(5, -2)$ are vertices of a parallelogram $ABCD$. What are the coordinates of point $C$?
(b) 3 points $A(0, 0)$, $B(2, 3)$, $D(4, 1)$ are vertices of a parallelogram $ABCD$. What are the coordinates of point $C$?
(c) 3 points $A(0, 0)$, $B(1, 5)$, $D(3, -2)$ are vertices of a parallelogram ABCD. What are the coordinates of the point $C$?
(d) Can you guess the general rule: if $A(0, 0)$, $B(b_1, b_2)$, $D(d_1, d_2)$ are 3 vertices of a parallelogram, what are coordinates of point $C$?

**9.** Find the coordinates of the midpoint of the segment $AB$ where $A = (3, 11)$, $B = (7, 5)$ [Hint: accurately draw the segment and visually find its midpoint]. Can you guess the general rule: if $M$ is a midpoint of a segment $AB$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$, what are the coordinates of the point $M$?

**10.** For each of the equations below, draw the graph, then draw the corresponding perpendicular line (going through the point (0, 0)) and then write the equation of the perpendicular line

(a) $y = 2x$ (b) $y = 3x$

(c) $y = -x$ (d) $y = -\frac{1}{2}x$

Can you guess the general rule: if the slope of a line is k, what is the slope of the perpendicular line?