

Math 6a/d: Homework 16

Deadline: Friday, February 5, 2021

Mathematical Games of Strategy

All games in this homework will have the same structure: two players take alternate turns. The player who takes the first turn is called *the first player*, and his or her opponent is *the second player*.

You'll be asked to look for a **winning strategy: a set of instructions that one of the players can follow in order to ensure his or her victory**. Firstly, these instructions should be *precise* and *complete*: whatever the opponents' move is, the player should be able to reply with a move according to the instructions. Secondly, the winning strategy should be *verifiable*: it should be possible to explain why this set of rules is certain to bring victory to the player.

Our toolbox of winning strategies so far contains two handy ideas: symmetry and complements.

Problems

1. Suppose you are playing two chess games on two boards at the same time. Both of your opponents are chess world champions. How should you organize the games and how should you play in order to win or have a draw in at least one of the games?
2. Terrence and Clarence are playing a game. They have 2 piles of candy, 23 pieces in each. One every turn, a player is allowed to take up to 5 candies from any single pile. (Skipping turns is not allowed.) The winner is the person who takes the last candy. Terrence goes first. Who has a winning strategy, and what is it?
3. Kevin and Deia are playing a game. They have 2 jars of cookies, one with 31 cookies and the other with 27. On every turn, a player is allowed to take up to 6 cookies from the same jar (a player has to take at least 1 cookie per turn). The person who takes the last cookie wins the game. Kevin goes first. Who has a winning strategy, and what is it?
4. Sophie and Michelle take turns placing rooks on the squares of the 8x8 chessboard in such a way that the rooks cannot attack one another (in this

- game, the color of the rooks is irrelevant.) The player who cannot place a rook loses the game. Sophie takes the first turn. Who has a winning strategy, and what is it?
5. Savir and Ben take turns placing 1×2 dominoes on a 10×10 chessboard. Each domino should be placed so as to fully cover two squares of the board; the dominoes cannot overlap. The player who cannot place a domino loses. Savir goes first. Who has a winning strategy, and what is it?
 6. Michael and Aish are playing a game. The first player writes a one-digit number on the board. After that, the players take turns adding digits to this number. They stop when the number reaches 16 digits. Michael wins if the resulting 16-digit number is divisible by 9; Aish wins otherwise. Aish goes first. Who has a winning strategy, and what is it?
 7. Aishwarya and Valentina are playing a game. They have a pile of 56 marbles. On a single turn, a player can take up to 8 marbles from the pile. The player who cannot make his move loses the game. Aishwarya goes first. Who has a winning strategy, and what is it?
 8. Rishika and Shagun are playing a game. They take turns breaking a rectangular chocolate bar into pieces. The bar is 6 squares wide by 10 squares long. On her turn, a player snaps an existing piece of chocolate into two smaller pieces (breaking along the line between squares). The player who cannot take a turn loses the game. Rishika goes first. Who has a winning strategy, and what is it?
 9. Amogh and Alex are playing a game. The number 1 through 10 are written on the board in a row. The players take turns inserting plus and minus signs between the numbers. After all nine signs have been placed, the resulting expression is evaluated; that is, the additions and subtractions are performed. The first player wins if the answer is even, the second wins if the answer is odd. Amogh goes first. Who has a winning strategy, and what is it?
 10. Masha and Tolya are playing a game on a board of size 5×2021 . On his turn, each player crosses out a 2×2 square that does not intersect with any square that has already been crossed out. (In this game, two squares intersect if they share at least one 1×1 square.). The player who cannot make such a move loses the game. Masha goes first. Who has a winning strategy, and what is it?