## BASIC PROBABILITY

Basic probability rule:

$$P(win) = \frac{\text{number of outcomes that meet my condition}}{\text{number of equally likely outcomes}}$$

For example, probability of drawing a spade card out of the standard deck is

$$P = \frac{13}{52} = \frac{1}{4}$$

## COMPLEMENT RULE

If probaility of some event is $P$ then the probability that this event will **not** happen is $1 - P$.
For example, if we draw a card from the deck then the probability that it is **not** a spade is $1 - \frac{1}{4} = \frac{3}{4}$.

## ADDITION RULE

$$P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$$

When A and B are mutually exclusive events (like getting 2 and getting 3 when rolling a die), then P(A AND $B) = 0$.

## PRODUCT RULE

If we do two trials (e.g., rolling a die twice), then the probability of getting result $A$ in the first trial and result $B$ is the second one is

$$P(\text{A, then B}) = P(A)P(B)$$

for independent events, i.e. if results of the second trial **do not depend** on the first one.

## EXAMPLE: TOSSING A COIN

**Question.** If toss a coin 10 times, what is the probability that all will be heads?
**Answer.** $\left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}}$ (using calculator, one can compute that it is $1/1024 \approx 0.001$, or $1/10$ of 1%).

**Question.** If toss a coin 10 times, what is the probability that all will be tails?
**Answer.** The same.

**Question.** If we toss a coin 10 times, what is the probability that **at least one** will be heads?
**Answer.** Unfortunately, there are very many combinations which give at least one heads. In fact, it is easier to say which combinations **do not** give at least one heads: there is exactly one such combination, all tails; probability of getting this combination is, as we computed, $1/2^{10} = \frac{1}{1024}$. The remaining combinations will give at least one heads; thus probability of getting at least one heads is $1 - \frac{1}{1024} = \frac{1023}{1024} \approx 0.999$.

When asked "Would you prefer to play for A or for B?" in a gambling game, it makes sense to calculate expected value, i.e. the average gain or loss of an event if the procedure is repeated many times. We can compute the expected value by multiplying each outcome by the probability of that outcome, then adding up the products. For example, in a game when A gets 10 dollars from B for getting 3 tails in a row when flipping a coin 3 times, and B gets 1 dollar from A in any other case, the expected value for the player A is 10x1/8 - 1x7/8 = 3/8. On the other hand the expected value for the player B is -3/8. You're on average better off to play for A.

1. We take the standard card deck and draw one card. What is the probability that the card will be
   (a) Queen of hearts
   (b) A queen or a hearts card
   (c) A red card
   (d) A picture card (a jack, queen, king, ace)
   (e) A picture card other than the queen of hearts

2. (a) What is the probability that when we toss a coin 4 times, there will be no heads?
   (b) A and B are playing the following game. They toss a coin 4 times; if there are no heads, A wins, and B pays him $10. Otherwise A loses and he pays $1 to B.
       Would you prefer to play for A or for B in this game?

3. (a) What is the probability that when we roll two dice, at least one will be a 6?
   (b) A and B are playing the following game. They roll two dice; if at least one is a 6, A wins, and B pays him $5. Otherwise A loses and he pays $1 to B.
       Would you prefer to play for A or for B in this game?

4. (a) What is the probability that if we roll 3 dice, all the numbers will be odd?
   (b) A and B are playing the following game. They roll 3 dice; if all numbers are odd, A wins, and B pays him $5. Otherwise A loses and he pays $1 to B.
       Would you prefer to play for A or for B in this game?

5. Supposing that there are equal chances of a boy or a girl being born, what is the probability that the first five babies born next Saturday morning at the St. Charles Hospital will be girls? That at least one of the five will be a girl?

6. In a certain club of 30 people, they are selecting a president, vice-president, and a treasurer (they all must be different people: no one is allowed to take two posts at once). How many ways are there to do this?

7. In a group of 100 students, 28 speak Spanish, 30 speak German, 42 speak French; 8 students speak Spanish and German, 10 speak Spanish and French, 5 speak German and French and 3 students speak all 3 languages. How many students do not speak any one of the three languages?
   [Note: when it says that 28 students speak Spanish, this includes the 8 who speak Spanish and German; similarly for all other combinations.]

8. How many whole numbers between 1–1000 are divisible by 3? by 5? by 15? are not divisible by either 3 or 5 or both?

9. (a) What is the probability that if we roll 2 dice, the sum will be at most 7?
   (b) A and B are playing the following game. They roll 2 dice; if the sum is at most 7, A wins, and B pays him $1. Otherwise A loses and he pays to B $1.
       Would you prefer to play for A or for B in this game?

10. (a) What is the probability that if we roll 3 dice, all the numbers will be different?
    (b) A and B are playing the following game. They roll 3 dice; if all numbers are different, A wins, and B pays him $2. Otherwise A loses and he pays to B $3.
        Would you prefer to play for A or for B in this game?