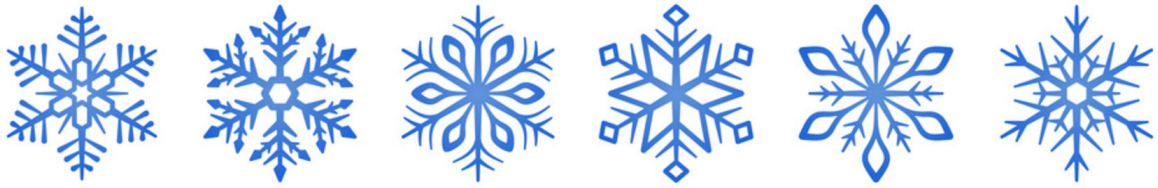


Math 6a/d: Homework 12

Deadline: Wednesday, January 6th, 2021

Factorial and Permutations

If we are choosing k objects from a collection of n so that a) order matters and b) no repetitions allowed, then there are

$$n(n\text{-}1)(n\text{-}2)\ldots \quad (k \text{ factors})$$

ways to do it. So the number of permutations of n objects taken k at a time is determined by the following formula that we discussed in class:

$$P(n, k) = {}_nP_k = n!/(n\text{-}k)!$$

1. If Ariel is cast a movie, Belle will refuse to be in the cast. Ariel is never cast in a movie where Jasmine is in the cast. And either Belle or Jasmine (or both) will certainly star in Disney's upcoming movie.

Based on all of this, can you explain why it is impossible that Ariel be in the cast of Disney's upcoming movie?

2. (a) Prove (by truth table or otherwise) that B OR A OR B is not equivalent to A.

(b) Prove (by truth table or otherwise) that B XOR A XOR B is equivalent to A.

3. Let A be the set of even integers and B be the set of multiples of 3.
   (a) Is it possible to find a set C such that $(A \cup B) \cup C = A$?
   (b) Is it possible to find a set C such that $(A \cup B) \cap C = A$?

4. Once upon a time there was a king who had many beautiful daughters. There were many dangerous tigers in his kingdom as well. A knight came to the king. He wanted to marry a princess. The king decided to test how clever the knight was. He offered the following test. There were three doors leading to three rooms. In one of

the rooms, there was a princess waiting for the knight. In another, there was a a hungry tiger. The third room was empty.

The king also placed the signs on the doors — but the sign on the door of princess's room was true, the sign on the tiger's door was false, and the sign on the door of the empty room could be either false or true. Here are the signs:

Room I	Room II	Room III
Room III is empty.	The tiger is in room I.	This room is empty.

Which door should the knight open?

5. (a) A palindromic number is a number that reads the same backward and forward. (For example, 13, 531 is palindromic.) How many 5-digit numbers are palindromic?

(b) How many 5-digit numbers are palindromic and consist of distinct digits?

(c) How many 5-digit numbers consist of distinct digits and end with 2?

(d) How many 5-digit numbers are odd and consist of distinct digits?

6. Tolya, who wrote an online computer game, plans to assign every user a unique password.

(a) His current plan is to have passwords that are 8 symbols long and are all made from lowercase letters a, b, and c. How many different passwords can Tolya generate?

(b) His friend Terrence suggests this modification: use 8-symbol passwords made from the letters a, b, and c, with exactly1 capital letter. How many times more passwords can Tolya generate now?

(c) Finally, Tolya decides that his passwords will be 9 symbols long, made from 1 digit (0 to 9) and from letters a, b, and c, with exactly 1 capital letter. How many times more passwords can he generate compared to the original plan?

7. (*Open problem, come up with as many solutions as you can, please do not google the control one*).

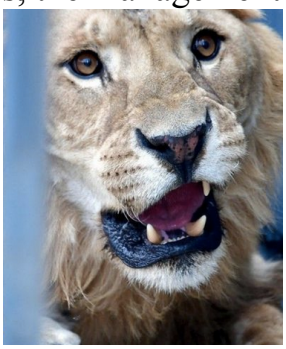
In large cities in Australia, there is a problem with small (and in fact, cute) animals - the possums. They eat young leaves,

buds, flowers and fruits on trees in city parks, leaving the trunk practically naked.

Sydney city officials were the first to sound the alarm and call on inventors for help. Even for the purpose of improving parks, they cannot destroy animals that are part of the unique nature of Australia. At the same time, the trees must be preserved.
Help solve this problem!

8. (*Open problem, come up with as many solutions as you can, please do not google the control one*).

In one zoo, in an effort to preserve rare species of animals and attract visitors, the management constantly enlarged the collection.



The number of inhabitants increased, and the cost of their maintenance increased every day. The zoo did not have enough money to feed the pets. The diet had to be cut, the animals were starving and sick. In order to receive the necessary money, the administration increased the cost of entrance tickets. But, to the great regret of the animals, the visitors considered the new price (twice as expensive) unreasonable. Fewer and fewer people visited the zoo, the animals became more and more hungry.

Out of despair, the management had the idea to sell some of the pets and lease the vacated territory to visiting amusement parks.
But this is a bad decision. Let's try to find a good one!

9. Can you come up with an example of a number that has 10 different factors?

10. Solve the encrypted problem. (The same letter always stands for the same digit, and different letters stand for different digits.)

$$\begin{array}{r} T\,A\,D \\ +\,T\,A\,T \\ \hline H\,A\,H\,A \end{array}$$