Math 6a/d: Homework 11

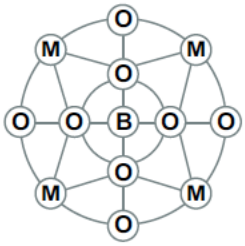
Deadline: Friday, December 18th, 2020

In the following problems you'll get to practice a few important concepts that we discussed at our two previous classes:

1. **Slot rule** (some people call it product rule or multiplication rule). The main character encounters a series of sets of choices to make for each slot. The choices at each decision point could be make **independently** from other choices (our towns-and-roads problems), or the choices that need to be made at each step depend on the previously made decisions (permutation problems).
2. The simplified formula to solve problems that fall into this category is this: *identify the slots, count the number of options for each slot, and multiply these numbers to calculate the answer.*
3. In **permutation problems** we have a certain number of different objects - three pieces of candy, or four letters of the alphabet - and we want to count the number of different ways to order them, or arrange them in a row. Here the choices that need to be made at each step **depend on the previously made decisions** and we should consider this when counting the number of options for each slot.
4. Often problems might look as if they are too complicated to be solved using the slot rule but don't fall into this. In all homework problems there is a way to split them into a few simpler problems that would look exactly like the ones you know for sure how to solve using the slot rule. The challenge is to find that way!
5. The final answer to a problem - the number of ways to do this or that - erases all traces of how the problem was solved. The answer is more informative and easier to understand if it is expressed as **numerical formulas**, not as number. But of course you are welcome to practice arithmetics and provide the final answer as well.

Enjoy!

1. In how many different ways can you trace the word BOOM on the drawing below?



2. While protecting his swamp, an ogre fights with four ironclad knights: Sir Allister, Sir Ballister, Sir Callister, and Sir Dallister. He intends to knock them down one by one, but he is not yet sure in what order. How many ways are there for the ogre to knock the knights down?

3. Gabby the Elf has 3 overalls of different colors: blue, green, and brown. He also has 5 different hats: 3 yellow and 2 red. Finally, he owns 6 different pairs of shoes: 2 yellow and 4 red. Gabby is selecting an outfit: a pair of overalls, a hat, and a pair of of shoes. In how many ways can he do it if he wants the color of his shoes to match the color of his hat.

4. Mr. and Mrs. Jones have 6 kids - 3 boys and 3 girls. Today, a photographer is taking their family's Christmas pictures.

   (a) In how many ways can the kids be seated in a row for a photo shot?

   (b) In how many ways can the kids be seated in a row so that all the girls are the left and all the boys are on the right?

   (c) In how many ways can the Jones kids be seated in a row so that boys and girls alternate?

   (d) All the girls must be sitting together, all the boys must be sitting together as well, and the parents must be either together in the center of one on each side.

5. A damaged robot named R2 remembers 5 digits only: 1, 2, 3, 4, 5.

   (a) How many 4-digit numbers can robot R2 write using these digits only?

   (b) How many 4-digit numbers with exactly 1 digit "3" can it write?

   (c) How many 5-digit numbers with exactly 2 digits "3" that are 1 apart?

   (d) How many 4-digit numbers with all digits different?

(e) How many 4-digit numbers with all digits different that start with an odd digit?

(f) How many 4-digit numbers with all digits different that end with an even digit?

(g) How many 5-digit numbers with alternating odd and even digits?

6. There are only 6 letters in the Martian alphabet. All Martian words are exactly 4 letters long.

(a) Suppose that any sequence of 4 letters is a valid Martian word. How many words are there in the Martian language?

(b) Suppose that any sequence of 4 non-repeating letters is a valid Martian word. How many words are there in the Martian language?

(c) Suppose that any sequence of 4 letters that has at least one repetition is a valid Martian word. How many words are there in the Martian language? (Hint: How can you use the results of (a) and (b) to figure out an easy way to solve (c)?)

7. Michael, Alex, Savir and Kevin are in charge of organizing a lottery for their school Christmas fundraiser. They plan to print 1000 lottery tickets and sell them at $1 per ticket. The tickets are numbered 1 to 1000. The winning tickets will be announced at the fundraiser. Right now the team is discussing the criteria for the winning tickets.

- Michael suggests that all tickets with numbers composed only of odd digits should win $7 per ticket (for example, 11, 3, 199, 375, 111).

- Alex proposes a different scheme: to award $8 for any ticket that ends with the digit 1 (for example, 31, 1, 191, 371, 111).

- Savir favors the idea that every ticket that contains exactly one digit 3 will get $3 (for example 3, 31, 345, 131, 293).

The three of them start to argue: each one thinks that his plan is the best (will collect the most money if all the tickets are sold). Kevin, who attends a math class at School Nova and worked hard on his combinatorics homework, does some calculations. Then he explains how much money each plan would bring. According to Kevin, which plan is the most profitable?

8*. 10 elves must form a circle for some dance. In how many ways can they do this? (consider any rotation of the circle as the same order)

9. Little Bear goes through a bucket of honey in 15 minutes. Father Bear gobbles up the same amount of honey twice as fast. How long would it take for the two of them to finish up one bucket of honey together?

10. (a) Come up with 4 positive integer numbers such that their sum is equal to their product.

(b) Come up with 1000 positive integer numbers such that their sum is equal to their product.