1. Four towns in Elbonia are connected by roads as shown in the figure. How many different routes can one take from A to C? (Assume that for this trip you are only allowed to travel in the direction of the arrows.)
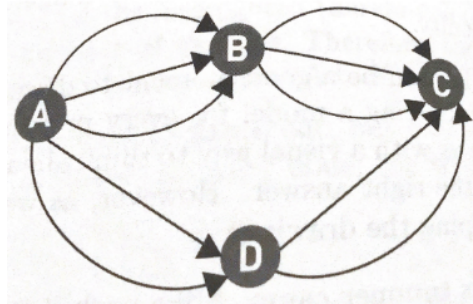
In the following problems **illustrate your solution (towns-and-roads or tree diagram style)**. Remember that just the answer doesn't give me a clue how you solved the problem, so make sure to write down all the numbers with corresponding operations.

2. Polly is talking parrot who speaks in 3-word sentences. A Polly's sentence always starts with a pronoun, which is followed by a verb, and then by a noun. Polly knows:

   - 2 pronouns: I and WE

   - 3 verbs: LOVE, WANT, and COOK,

   - 4 nouns: FOOD, CRACKER, FRIEND, and SHMOLLAR

Polly's friend Dolly the parrot can talk as well. A Dolly's sentence always starts with an adjective, which is followed by a noun, and then by a verb:

   - 3 adjectives: HAPPY, HUNGRY, and LONELY,

   - 2 nouns: PARROT and CROCODILE,

   - 3 verbs: SINGS, CRIES, and WORKS.

(a) How many different phrases can polly the parrot say?

(b) How many different sentences can Dolly the parrot say?

(c) Polly and Dolly are creating a two-phrase story. Each parrot contributes a sentence. How many different stories can they come up with?

3. For his summer camp, Michael packed two pairs of shoes, two pairs of shorts, and six T-shirts. In how many ways can he choose an outfit for a day at the camp? (Michael's outfit consists of a pair of shorts, a T-shirt, and a pair of identical shoes.)

4. For her summer camp, Aishwarya packed four pairs of shoes, two skirts, five blouses, and three dresses. In how many ways can she choose an outfit for a day at the camp? (Aishwarya's outfit consists either of a skirt, a blouse and a pair of identical shoes or of a dress and a pair of identical shoes.)

5. Deia has a bag of green marbles, a bag of red marbles and a bag of blue marbles. In how many different patterns can she place two of these marbles in a row? Three marbles? Four marbles?

For the following problems I don't insist that you illustrate your solution (it may be still very helpful!). However, make sure that you explain what you did to come up with the answer.

6. The alphabet of the planet Xork has four letters only: A, B, C and D. The planet is divided into two kingdoms: Talkers and Chatterers.

(a) In the kingdom of Chatterers, a first name can be any three-letter word (that is, letter combination) composed of the letters A, B and C. How many different first names do they have in this kingdom?

(b) In the same kingdom, a last name can be any four-letter word composed of letters A, B, C and D, so long as it ends in D. How many different last names do they have in this kingdom?

(c) In the kingdom of Talkers, a first name can be any word *of up to three letters* composed of letters A, B and C. How many different first names do they have in this kingdom?

(d) In the same kingdom, a last name can be any four-letter word composed of letters A, B, C and D, so long as it has only one D in it, and this D is either the first or the last letter. How many different last names do they have in this kingdom?

7. Prince Terrence is on a quest to free Princess Sophie, who has been imprisoned in the castle. The castle door has a simple digital lock with ten buttons, numbered 0 to 9. The door is guarded by a hungry dragon, Alex, who likes hot dogs. The door lock can be opened by typing a secret 4-digit code, and Alex can be distracted by hot dogs. combination. It takes 1 seconds for Prince Terrence to try out a single 4-digit combination, and it takes 20 seconds for Alex to gulp down a single hot dog. After Terrence opens the lock, it will take him one minute to fetch Sophie and fly off on his magic carpet.

(a) How many hot dogs should Terrence pack for the quest if he wants to fly out of the castle alive and with Sophie?)Terrence should have enough hot dogs to keep Alex at bay even in the worst case: when the secret code will be the last possible combination that he tries.)

(b) Suppose that Terrence knows in advance that the secret 4-digit code is composed of odd digits only. How many hot dogs will he need now?

(c) Suppose that Terrence knows in advance that the secret 4-digit code is composed of odd digits only and has exactly one digit 5 in it. How many hot dogs will he need in this case?

8. A long time ago on the Island of Knights and Knaves there lived three friends, all mighty warriors. Their names were Clarence, Kevin and Amogh. Two of them were knaves, and one was a knight. The friends kept their affiliations secret. One of their great deeds was a battle with a terrible dragon that was terrorizing the kingdom. Not much is known about this battle except that the dragon was killed by a knight. In a recently discovered old letter, Clarence states that Amogh had killed the dragon. Can you tell us who actually killed the dragon?

9. Consider the following sets:

Z- al whole numbers (positive and negative)

N - all positive whole numbers

R - all numbers

Q - all rational numbers (i.e., those that can be written as a fraction)

Order them from smallest to largest, so that each set is a subset of the next one.

10*. Let A be the set of all ordered pairs of numbers (x,y) such that 0<x<1, and B be the set of all ordered pairs of numbers (x, y) such that 0<y<1.

    (a) Is (0.5, 1) in A? Is it in B?

    (b) Determine where the following points are in AUB: (0.1, 0.2), (0.1, 2.5), (1.1, 1.1), (-2, 0.5).