

Mathematical Auction Problems

Each problem is worth 100 shmollars.

Problem 1. Using five times the digit 2, represent as many consecutive natural numbers as possible, starting from 1. You can use the arithmetic operations +, -, x, ÷, and parentheses. The same operation can be used several times. You are not required to use all four operations.

Examples:

$$1 = (2 - 2/2) \times 2/2, \quad 2 = 22 - 22 + 2.$$

A team has a stronger solution for this problem if it is able to continue the list starting from where the previous team stopped.

Problem 2. Cut a circle with seven straight lines so as to get as many triangular pieces as possible. “Triangles” with curved sides don’t count, nor do cut triangles. (That is, do not count triangles made of smaller triangles or polygons.)

A team has a stronger solution for this problem if it is able to present a drawing that has more triangles.

Problem 3. Cut 7 round pizzas into wedges in such a way as to be able to equally divide these pizzas between 8 people. Are as few cuts as possible. (A cut is a straight line that starts and ends at an outer edge of a pizza. Everybody should get the same share, and there should be no leftovers. Different people can get a different assortment of slices as long as they add up to equal shares.)

A team has a stronger solution for this problem if it is able to cut the pizzas using fewer cuts.

Problem 4. Ali the Trader has a gold chain with 20 links. (The chain is not fastened into a loop.) During one of his expeditions, Ali hires a guide for a 20-day trip through the desert. The guide asks him for one gold link per day as payment - and he wants to be paid at the end of each day.

In order to make the daily payments, Ali has to open several links of his chain. Since unfastening a link costs money, Ali does not want to open too many links. The good news is that the guide is willing to trade links with Ali to make things even.

For example, if Ali pays with a single link on day one, he can give the guide a two-link chain on day two and get the single link back.

What is the smallest number of links Ali the Trader would need to open to be able to pay his guide for the trip?

A team has a stronger solution for this problem if it is able to present a solution with fewer open links.