MATH 6 A/D
HOMEWORK 7: SETS
DEADLINE: NOVEMBER 13, 2020

SETS

Describing Sets. By word *set*, we mean any collection of objects: numbers, letters, ... Most of the sets we will consider will consist either of numbers or points in the plane. Objects of the set are usually referred to as *elements* of this set.

Sets are usually described in one of two ways:

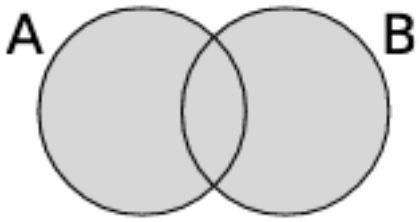
- By explicitly listing all elements of the set. In this case, curly brackets are used, e.g. $\{1, 2, 3\}$.
- By giving some conditions, e.g. "set of all numbers satisfying equation $x^2 > 2$". In this case, the following notation is used: $\{x \mid \dots\}$, where dots stand for some condition (equation, inequality, ...) involving x, denotes the set of all x satisfying this condition. For example, $\{x \mid x^2 > 2\}$ means "set of all x such that $x^2 > 2$".

Members of sets. Sometimes we might have to say whether the element belongs to the set or not. In this case the following notation is used:

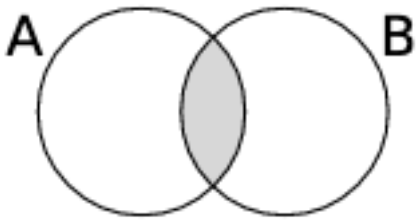
- $x \in A$ means "x is in A", or "x is an element of A"
- $x \notin A$ means "x is not in A"

Set Operations. There are several operations that can be used to get new sets out of the old ones:

- $A \cup B$: *union* of A and B. It consists of all elements which are in either A or B (or both):
$$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}.$$



- $A \cap B$: *intersection* of A and B. It consists of all elements which are in both A and B:
$$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}.$$



- $\overline{A}$: *complement* of A, i.e. the set of all elements which are not in A: $\overline{A} = \{x \mid x \notin A\}$.

Intervals. The following notations are used when we talk about intervals on the number line. Intervals may have end points included or excluded: [and] represent that the end point is included, while (and) indicate that the end point is excluded.

$[a, b] = \{x \mid a \le x \le b\}$ is the interval from a to b (including endpoints),
$(a, b) = \{x \mid a < x < b\}$ is the interval from a to b (**not** including endpoints),
$[a, \infty) = \{x \mid a \le x\}$ is the half-line from a to infinity (including a),
$(a, \infty) = \{x \mid a < x\}$ is the half-line from a to infinity (**not** including a)

Homework

1. Consider the operation NOR which is just the opposite of OR: it returns T only if both A and B are F. Using only the component NOR, see if you can create circuits equivalent to AND, OR, and NOT as we did in class with the operation NAND.
2. Using only AND, NOT, and OR, produce a three-input AND circuit, i.e., the output is F unless all three inputs are a T. (You do not have to use all three circuit elements.)
3. Using only AND, NOT, and OR, produce a three-input OR circuit, i.e., the output is T if any of the inputs is T.
4. If Terrence comes to a party, Alex will not come. Terrence never comes to a party where Kevin comes. And either Alex or Kevin (or both) will certainly come to the party.

   Based on all of this, can you explain why it is impossible that Terrence comes to the party?
5. Let
   - A=set of all students who know Chinese
   - B=set of all students who know Romanian
   - C=set of all students who know Russian

   Describe in words the following sets:
   (a) $A \cap B$ (b)$A \cup (B \cap C)$ (c) $(A \cap B) \cup (A \cap C)$ (d) $C \cap \overline{A}$.
6. Let us take the usual deck of cards. As you know, there are 4 suits, hearts, diamonds, spades and clubs, 13 cards in each suit.

   Denote:
   - H=set of all hearts cards
   - Q=set of all queens
   - R=set of all red cards

   Describe by formulas (such as $H \cap Q$) the following sets:
   - all red queens
   - all black cards
   - all cards that are either hearts or a queen
   - all cards other than red queens

   How many cards are there in each set?
7. In a class of 25 students, 10 students know French, 5 students know Chinese, and 12 know neither. How many students know both Chinese and French?
8. Draw the following sets on the number line:
   (a) Set of all numbers x satisfying $x \leq 2$ and $x \geq -5$;
   (b) Set of all numbers x satisfying $x \leq 2$ or $x \geq -5$
   (c) Set of all numbers x satisfying $x \leq -5$ or $x \geq 2$
9. For each of the sets below, draw it on the number line and then describe its complement:
   (a) $[0, 2]$ (b) $(-\infty, 1] \cup [3, \infty)$ (c) $(0, 5) \cup (2, \infty)$
10. A little gnome was born in the year 1122. On every birthday, he receives a precious stone from his grandma: either a diamond or a ruby. His grandma presents him with a diamond on those years when the gnome's age is a factor of that year's number. She gives him rubies on all other birthdays. For example, the gnome received a diamond on his first birthday (since 1123 is divisible by 1), on his second birthday (since 1124 is divisible by 2), on his third birthday(since 1125 is divisible by 3) and so on. How old was the gnome when he received his last birthday diamond? (Assume that both little gnomes and their grandmas live thousands and thousands of years.)