MATH 6 A/D
HOMEWORK 6: LOGIC PUZZLES
DEADLINE: NOVEMBER 6, 2020

BASIC LOGIC OPERATIONS

For you convenience, here is the list of logic operations we have seen so far:

- NOT A: true if A is false, false if A is true
- A AND B: true if both A and B are true, false otherwise
- A OR B: true if at least one of A and B is true, false otherwise
- A XOR B: true if exactly one of A and B is true, false otherwise

IF

- Statement $A \Rightarrow B$ is false when A is true and B is false; in all other situations $A \Rightarrow B$ is true.
- We showed that $A \Rightarrow B$ and $B \Rightarrow A$ are not equivalent: it is possible that one statement is true and the other is false.

For the truth tables of these operations refer to the previous homework description or classwork Jamboard posted in the stream on Google Classroom.

HOMEWORK

1. Simplify the following expressions.

(a) $\dfrac{6^5 \times 2^5}{3^5 \times 2^2} =$

(b) $(5^3)^3 =$

(c) $(7^2 \times 7^3)^2 =$

(d) $2^{-2} =$

2. Prove the contrapositive rule that $A \implies B$ is equivalent to (NOT B) $\implies$ (NOT A) by building their truth tables (thus, ''If you do not see my mirrors, then I do not see you" is equivalent to "If I see you, then you see my mirrors").

3. On the island of knights and knaves, you meet two inhabitants: Amogh and Sophie. Amogh tells you that Sophie is a knave. Sophie says, "Neither Amogh not I are knaves". **Make a truth table** for this problem and find out who is who?

4. On the island of knights and knaves, you meet two inhabitants, Terrence and Kevin. Terrence says, "Kevin is a knave". Kevin says, "Terrence is a knave". **Make a truth table** for this problem and find out who is who?

5. Write truth tables for formulas A AND(B OR C) and (A AND B) OR C (remember that there will be 8 rows in the table). Are these formulas equivalent (i.e., do they always give the same answer)?

6. Let us consider a new logical operation, called NAND, which is defined by the following truth table:

A	B	A NAND B
T	T	F
T	F	T
F	T	T
F	F	T

(a) Show that A NAND B is equivalent to NOT(A AND B) (this explains the name: NAND is short for "not and").

(b) Show that A NAND A is equivalent to NOT A.

(c) Write the truth table for (A NAND B) NAND(A NAND B).

7. A particular musical elephant enjoys dancing, but only if it is wearing purple. Observing this elephant, I take the following notes:

D = *the elephant is dancing*

P = *the elephant is wearing purple*
The elephant dances only when wearing purple. It sometimes naps, no matter the color it is wearing.
From this, conclude whether the following statements are true or false:
(a) $D \implies P$
(b) $P \implies D$

In the next two problems, try yourself as a logic detective. You need to a)write the obvious conclusion from given statements; and b)justify the conclusion, by writing a chain of arguments which leads to it. It may help to write the given statements and conclusion by logical formulas (denoting the statements which are used by letters $A, B, \dots$ connected by logical operations OR, AND, , . . .).

8. If today is Thursday, then Jane's class has library day. If Jane's class has library day, then Jane will bring home new library books. Jane brought no new library books. Therefore,. . .

9. Here is one of Lewis Carrol's puzzles. Some of you may know him as the author of *Alice in Wonderland* and *Through the Looking Glass*. However, he was also a mathematician, and invented a huge number of logical puzzles.
All hummingbirds are richly colored..
No large birds live on honey.
Birds that do not live on honey are dull in color.
Therefore,. . .

10. (AMC) A company sells detergent in three different sized boxes: small (S), medium (M) and large (L). The medium size costs 50% more than the small size and contains 20% less detergent than the large size. The large size contains twice as much detergent as the small size and costs 30% more than the medium size. Rank the three sizes from best to worst buy.

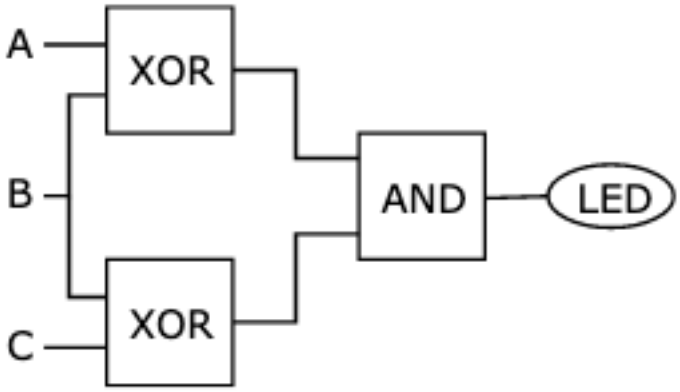
COMPUTER CIRCUITS AND LOGIC

Please watch the following video showing that computer circuits reproduce logical operations: https://www.youtube.com/watch?v=ZoqMiFKspAA (each of the inputs and outputs can have voltage 0 or a positive voltage). The usual convention is

- Positive voltage=true (T) = 1
- Zero voltage=false (F) = 0

Then one can relatively easily construct AND, NOT,. . . circuits, and combining them, more complicated circuits.

***11.** The diagram below shows some circuit constructed of 3 logical circuits (each with two inputs and one output; we draw them so that the inputs are on the left and the output, on the right). Can you determine for which values of inputs the LED will light up? Refer to our classwork Jamboard posted in Google Classroom for the truth table of XOR (problem 11 from Homework 5). [Hint: this is the same a writing a truth table for some formula....]



Note: the wires connecting each of the chips and LED to the power source are not shown.