

**MATH 6A/D**  
**HOMEWORK 4: KNIGHTS AND KNAVES**  
**DEADLINE: OCTOBER 23, 2020**

Yesterday we discussed the definition of propositions in formal logic (declarative statements that are either true or false). They are basic objects (like numbers in arithmetic) in this new formal language that we are learning. Next time we'll talk about some operations with propositions (similar to addition or multiplication on numbers in arithmetic).

In this homework you are asked again to work on a few more knaves and knights problems. I realise that my explanation yesterday might have been confusing for some of you. If it is so, please watch a couple of videos I posted with the assignment on Google Classroom. In my opinion, they explain how to solve this type of problems well. In addition to your answer, I should see some solution, either a table or words explaining your thought process.

Enjoy!

1. Solve the following equations:

$$(a) 2x - 22 = 3(1 - x) \quad (b) 1 - \frac{2}{7}x = \frac{1}{7}x \quad (c) 1 - 8(1 - x) = 7x - 8$$

2. Alex says that his older brother Kevin ate at least 12 cookies, while Kevin himself is sure that he ate no more than 13. It is known that Kevin consumed twice as many cookies as Alex and that both of them always tell the truth. How many cookies did Kevin eat?
3. Suppose the Big Bad Wolf declares, "There are at least 10 pigs in this house." If you know that the Wolf is a liar, what would be a true statement about the pigs in the house?
4. While visiting the Knights and Knaves Island, you meet two islanders, Clarence and Terrence. Clarence tells you that at least one of the two is a knave. Are Clarence and Terrence knights or knaves?
5. On the island of knights and knaves, you meet two inhabitants: Valentina and Rishika. Valentina tells you that Rishika is a knave. Rishika says, "I and Valentina are knights." So who is a knight and who is a knave?
6. Thirty children came to a party. Out of any 12 of them, at least one is a boy. Out of any 20 of them, at least one is a girl. How many boys and how many girls are there at this party?
7. On the island of Knights and Knaves, you meet two inhabitants: Ben and Savir. Ben says, "I and Savir are both knights or both knaves." Savir claims, "Only a knave would say that Ben is a knave." [Hint: first, rewrite Savir's claim in an easier to understand form.]
8. While visiting the Knights and Knaves Island, you come to a party. Every single person at this party tells you that there are some knaves in the room. What is really happening? How many knights and knaves are at this party? (Remember that there can also be tourists, such as you. Tourists sometimes lie and sometimes tell the truth.)
9. Aishwarya and Michelle are from the Island of Knights and Knaves. One of them is a knave and the other is a knight. Aishwarya claims that 2 rubies are more expensive than 3 sapphires. Michelle says that 3 rubies are more expensive than 4 sapphires. Is it the case that 12 rubies are more expensive than 18 sapphires?
10. Aish, Deia, Michael and Sophie are all inhabitants of the Island of Knights and Knaves. Aish claims that Deia is a knave. Sophie states that Aish is a knave. Michael declares that both Aish and Deia are knaves. Michael also states that Sophie is a knave as well. Who is what? Justify your answer.

- \*11. (The starred problems are those which are more difficult and thus are optional.) On the Island of Knights and Knaves, knights and knaves live in separate villages. However, they often visit their friends in other villages.

This week the island newspaper announced a contest: to come up with a single yes/no question that would allow a tourist to figure out whether he is in a knights' village or in a knaves' village. The question should be such that it can be addressed to anyone the tourist sees. It is not known in advance whether this local is a knight or a knave and whether he is a resident of this village or not.

Tolya, the SchoolNova student won the prize. What was his question?

- \*12. The country of FarAwaynia is composed of several states; it also has several political parties. Once, a group of FarAwanian politicians got together for a dinner. It is known that the group contained people from at least two different states and from at least two different parties. Prove that there were two politicians at the dinner that represented different states and belonged to different parties.