

MATH 6
HOMEWORK 3: KNIGHTS AND KNAVES
DEADLINE IS WEDNESDAY, OCTOBER 14, 2020

Many of the questions of this assignment refer to the famous (among logic puzzle fans) island of Knights and Knaves. On this island, there are two kinds of people: Knights, who always tell the truth, and Knaves, who always lie. Unfortunately, there is no easy way of knowing whether a person you meet is a knight or a knave. . .

Copyright notice: a lot of these problems come from books of Raymond Smullyan. If you liked them, get his books in the library and you will find there many more puzzles of the same sort. I would especially recommend *The Lady or the Tiger?* and *The Riddle of Scheherazade*. You can also find a number of such puzzles online at <http://philosophy.hku.hk/think/logic/puzzles.php>.

1. Find the greatest common divisor and least common multiple of 132 and 90.
2. Solve the following equation: $5 - 2(1 - x) = 9$.
3. You meet two inhabitants: Peggy and Zippy. Peggy tells you that 'of Zippy and I, exactly one is a knight'. Zippy tells you that only a knave would say that Peggy is a knave.
Can you determine who is a knight and who is a knave?
4. You meet two inhabitants: Marge and Zoey. Marge says, 'Zoey and I are both knights or both knaves.' Zoey claims, 'Marge and I are the same.'
Can you determine who is a knight and who is a knave?
5. You meet two inhabitants: Ted and Zeke. Ted claims, 'Zeke could say that I am a knave.' Zeke claims that it's not the case that Ted is a knave.
Can you determine who is a knight and who is a knave?
6. You meet two inhabitants: Ned and Zoey. Zed says that it's false that Zoey is a knave. Zoey claims, 'I and Ned are different.'
Can you determine who is a knight and who is a knave?
7. You meet two inhabitants: Sue and Marge. Sue says that Marge is a knave. Marge claims, 'Sue and I are not the same.'
Can you determine who is a knight and who is a knave?
8. On the island of knights and knaves, you meet two inhabitants: Zoey and Mel. Zoey tells you that Mel is a knave. Mel says, "Neither Zoey nor I are knaves." So who is a knight and who is a knave?
9. You meet two inhabitants: Betty and Peggy. Betty tells you that Peggy is a knave. Peggy tells you, 'Betty and I are both knights.'
Can you determine who is a knight and who is a knave?
10. You meet two inhabitants: Zed and Peggy. Zed says that Peggy is a knave. Peggy tells you, 'Either Zed is a knight or I am a knight.'
Can you determine who is a knight and who is a knave?
11. You meet two inhabitants: Zed and Alice. Zed tells you, 'Alice could say that I am a knight.' Alice claims, 'It's not the case that Zed is a knave.'
Can you determine who is a knight and who is a knave?
12. While visiting the Knights and Knaves Island, you pass a beautiful garden where three islanders, Sam, Bob, and Tom are watching the sunset. You ask Sam, "Are you a knight or a knave?" Sam is shy; you cannot hear his quiet answer. So you ask Bob, "What did Sam say?" Bob answers, "He said that he is a knave." "Don't trust Bob! Bob is a knave!" screams Tom.
Can you decide whether Bob and Tom are knights or knaves?
13. A traveler to the island of Knights and Knaves meets a group of five people (call them A, B, C, D, E).

A says: "exactly one of us is a Knight"

B says: "exactly two of us are Knights"

C says: "exactly three of us are Knights"

D says: "exactly four of us are Knights"

E says: "all five of us are Knights"

Can you find out which of them are Knights?

14. You are in a maze on the island of knights and knaves. There are two doors: you know that one leads to freedom and one leads to certain doom. There are two guards nearby, and you happen to know that one is a knight and one is a knave, but you don't know who is who. They allow you to ask one of them a single question before you choose a door — what do you ask?
15. * Johnny decided to make a Rubik's Cube. He took a $3 \times 3 \times 3$ wooden cube and painted each of the six faces in a different color. After that, he sawed this cube into 27 small $1 \times 1 \times 1$ cubes. (It took him 6 cuts total - two horizontal ones, two vertical ones parallel to the front face, and two vertical ones parallel to the side face.)
 - (a) How many small cubes have exactly three painted faces? Two? One? How many cubes have no painted faces at all?
 - (b) Johnny's friend Michael says that Johnny should have stacked the parts of the cube together while sawing. This way, he would have achieved the same result with fewer than 6 cuts. Is Michael correct? Either show how to get 27 small cubes with fewer than 6 cuts, or explain why Michael is wrong.