MATH 6
PROBABILITIES: LET'S PLAY CASINO!

Basic probability

Basic probability rule:

$$P(win) = \frac{\text{number of winning outcomes}}{\text{total number of possible outcomes}}$$

For example, probability of drawing a spade card out of the standard deck is

$$P = \frac{13}{52} = \frac{1}{4}$$

Complement rule

If probability of some event is P then the probability that this event will **not** happen is $1 - P$.

For example, if we draw a card form the deck then the probability that it is **not** a spade is $1 - \frac{1}{4} = \frac{3}{4}$.

Product rule

If we do two trials (e.g., rolling a die twice), then the probability of getting result A in the first trial and result B is the second one is

$$P(\text{A, then B}) = P(A)P(B)$$

if results of the second trial **do not depend** on the first one.

Example: tossing a coin

Question. If toss a coin 10 times, what is the probability that all will be heads?

Answer. $\left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}}$ (using calculator, one can compute that it is $1/1024 \approx 0.001$, or 1/10 of 1%).

Question. If toss a coin 10 times, what is the probability that all will be tails?

Answer. The same.

Question. If we toss a coin 10 times, what is the probability that **at least one** will be heads?

Answer. Unfortunately, there are very many combinations which give at least one heads. In fact, it is easier to say which combinations **do not** give at least one heads: there is exactly one such combination, all tails; probability of getting this combination is, as we computed, $1/2^{10} = \frac{1}{1024}$. The remaining combinations will give at least one heads; thus probability of getting at least one heads is $1 - \frac{1}{1024} = \frac{1023}{1024} \approx 0.999$.

1. We take the standard deck of cards and draw one card. What is the probability that the card will be

(a) Queen of hearts
(b) Either a queen or a hearts card
(c) A red card
(d) A face card (a jack, queen, king,ace)
(e) A face card other than the queen of hearts

2. (a) What is the probability that when we toss a coin twice, we will get 2 heads?
(b) A and B are playing the following game. They toss a coin twice; if both tosses are heads, A wins, and B pays him \$4. Otherwise A loses and he pays to B \$1.
Would you prefer to play for A or for B in this game?

3. (a) What is the probability that when we toss a coin 4 times, there will be no heads?
(b) A and B are playing the following game. They toss a coin 4 times; if there are no heads, A wins, and B pays him \$10. Otherwise A loses and he pays to B \$1.
Would you prefer to play for A or for B in this game?

4. Suppose I have a standard die (6 faces on a cube, numbered 1 through 6).
(a) What is the probability that when we roll the die once, the number will be less than 5?
(b) What is the probability that when we roll the die once, the number will be less than 7?
(c) What is the probability that when we roll the die twice, at least one result will be a 6?
(d) What is the probability that when we roll the die twice, at least one result will be a 7?
(e) What is the probability that when we roll the die three times, all the results will be odd?

5. Let $A = \{1, 2, 3\}$ be the set of the numbers 1, 2, 3.
(a) Two numbers are randomly chosen from A, one after the other (repeats are allowed). What is the chance that both numbers are the same?
(b) Two numbers are randomly chosen from A, one after the other. What is the chance that they will be in strictly increasing order? (*Strictly increasing* means the second number must be greater than the first, they are not allowed to be equal.)

6. In the Great Amphibian Parliament, there are 40 Members of Parliament (MPs). 20 of them speak Toadish, 14 speak Salamander, and 12 speak Newt; 4 speak both Toadish and Salamander, 6 speak Salamander and Newt, and 5 speak Newt and Toadish; 2 speak all three languages. How many MPs do not speak any one of the three languages? [Note: when it says that 20 MPs speak Toadish, this includes the 4 that speak Toadish and Salamander; similarly for all other combinations.]

7. How many whole numbers between 1–1000 are divisible by 3? by 5? by 15? are not divisible by either 3 or 5?

8. (AMC) From a regular octagon, a triangle is formed by connecting three randomly chosen vertices of the octagon. What is the probability that at least one of the sides of the triangle is also a side of the octagon?

9. (AMC) A fair 6-sided die is rolled twice. What is the probability that the first number is greater or equal than the second one?