# MATH 6
# HANDOUT 5: SETS

## Sets

By the word *set*, we mean any collection of objects: numbers, letters,.... Most of the sets we will consider, consist either of numbers or points in the plane. Objects of the set are usually referred to as *elements* of this set.

Sets are usually described in one of two ways:

- By explicitly listing all elements of the set. In this case, curly brackets are used, e.g. $\{1, 2, 3\}$.
- By giving some conditions, e.g. "set of all numbers satisfying equation $x^2 > 2$". In this case, the following notation is used: $\{x \mid \dots\}$, where dots stand for some condition (equation, inequality, ...) involving $x$, denotes the set of all $x$ satisfying this condition. For example, $\{x \mid x^2 > 2\}$ means "set of all $x$ such that $x^2 > 2$".

Other notation:

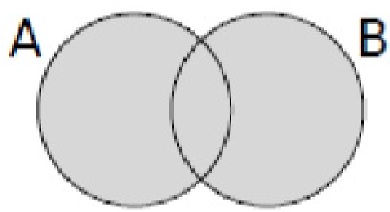
$x \in A$ means "$x$ is in $A$", or "$x$ is an element of $A$"

$x \notin A$ means "$x$ is not in $A$"

$\varnothing$ is the empty set, or set which contains no elements. It is sometimes useful for the same reasons it is useful to have a notation for number 0.

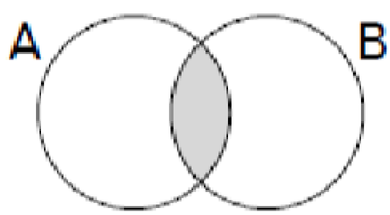
$A \cup B$: union of $A$ and $B$. It consists of all elements which are in either $A$ or $B$ (or both):

$$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}.$$




$A \cap B$: intersection of $A$ and $B$. It consists of all elements which are in both $A$ and $B$:

$$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}.$$




$\overline{A}$: complement of $A$, i.e. the set of all elements which are not in $A$: $\overline{A} = \{x \mid x \notin A\}$.

$|A|$: number of elements in a set $A$ (if this set is finite)

**1.** If Al comes to a party, Betsy will not come. Al never comes to a party where Charley comes. And either Betsy or Charley (or both) will certainly come to the party.

Based on all of this, can you explain why it is impossible that Al comes to the party?

**2.** Let $A = [1, 3] = \{x \mid 1 \le x \le 3\}$, $B = \{x \mid x \ge 2\}$, $C = \{x \mid x \le 1.5\}$. Describe $\overline{A}$, $\overline{B}$, $\overline{C}$, $A \cap B$, $A \cap C$, $A \cap (B \cup C)$, $A \cap B \cap C$.

**3.** Draw the following sets on the number line:
(a) Set of all numbers $x$ satisfying $x \le 2$ and $x \ge -5$;
(b) Set of all numbers $x$ satisfying $x \le 2$ or $x \ge -5$
(c) Set of all numbers $x$ satisfying $x \le -5$ or $x \ge 2$

**4.** (a) Using Venn diagrams, explain why $\overline{A \cap B} = \overline{A} \cup \overline{B}$. Does it remind you of one of the logic laws we had discussed before?
(b) Do the same for formula $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

**5.** In this problem, we denote by $|A|$ the number of elements in a finite set $A$.
(a) Show that for two sets $A, B$, we have $|A \cup B| = |A| + |B| - |A \cap B|$.
*(b) Can you come up with a similar rule for three sets? That is, write a formula for $|A \cup B \cup C|$ which uses $|A|, |B|, |C|$, $|A \cap B|$, $|A \cap C|$, $|B \cap C|$.

**6.** In a class of 33 students, 12 are girls, 10 play soccer, and 10 play chess. Moreover, it is known that 6 of the soccer players are girls, that 2 of the chess players also play soccer, and that there is exactly one girl who plays both chess and soccer. Finally, 4 girls play neither soccer nor chess. Can you figure out how many boys play soccer? chess? both? neither?

**7.** (AMC) For the game show Who Wants To Be A Millionaire?, the dollar values of each question are shown in the following table (where K = 1000).

| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Value | 100 | 200 | 300 | 500 | 1K | 2K | 4K | 8K | 16K | 32K | 64K | 125K | 250K | 500K | 1000K |

Between which two questions is the percent increase of the value the smallest?

**8.** (AMC) Which of the following has the largest shaded area?