MATH 6 A/D
HOMEWORK 5: TRUTH TABLES AND LOGIC LAWS
DEADLINE: OCTOBER 30TH, 2020

TRUTH TABLES

Logical variables A, B, C...: represent propositions, take value True (T) or False (F).

Basic logic operations:

NOT (for example, NOT A or $\neg A$): true if A is false, and false if A is true.

AND (for example A AND B or $A \wedge B$): true if both A, B are true, and false otherwise

OR (for example A OR B or $A \vee B$): true if at least one of A, B is true, and false otherwise. Sometimes also called "inclusive or" to distinguish it from the "exclusive or" described below.

As in usual algebra, logic operations can be combined into more complicated formulas, e.g. (A OR B) AND C (can be also written $(A \vee B) \wedge C$).

Truth tables: If we have a logical formula involving variables $A, B, C, \ldots$, we can make a table listing, for every possible combination of values of $A, B, \ldots$, the value of our formula. For example, the following is the truth tables for OR and AND:

A	B	A OR B
T	T	T
T	F	T
F	T	T
F	F	F

A	B	A AND B
T	T	T
T	F	F
F	T	F
F	F	F

Truth tables are useful in solving the problems about knights and knaves. Here is a typical problem: on the island of knights and knaves you meet two inhabitants, Alice and Ben. Alice tells you, 'I am a knight or Ben is a knave.' Ben tells you, 'Of Alice and I, exactly one is a knight.' We can use logical variables A = "Alice is a knight" and B = "Ben is a knight" and solve the problem by making the following table:

A	B	A is a knight or B is a knave	Of A and B, exactly one is a knight
T	T	T	F
T	F	T	T
F	T	F	T
F	F	T	F

From this table we see that only the third row is valid and conclude that Alice must be a knave and Ben must be a knight.

HOMEWORK

1. What is the LCM and GCD for 26 and 1001?

2. Solve the equation $20 - 3(x - 1) - (x + 1) = 6$

3. While visiting the Knights and Knaves Island, you meet two islanders, Clarence and Terrence. Clarence tells you that at least one of the two is a knave. **Make a truth table** for this problem and find out who is who?

4. On the island of knights and knaves, you meet two inhabitants: Valentina and Rishika. Valentina tells you that Rishika is a knave. Rishika says, "I and Valentina are knights." **Make a truth table** to figure out who is a knight and who is a knave?

5. On the island of knights and knaves you can meet 3 kinds of people:

knights, who always tell the truth

knaves, who always lie

tourists, who sometimes lie and sometimes tell the truth

On that island, you meet 3 people, A, B, and C, one of whom is a knight, one a knave, and one tourist (but not necessarily in that order). They make the following statements:

A: I am a tourist

B: That is true

C: I am not a tourist

What are A, B, and C?

6. Many trucks carry the message: "If you do not see my mirrors, then I do not see you". Can you rewrite it in an equivalent form without using the word "not"?

Define a new logical operation IF (as in "if A, then B; written $A \implies B$): if A is false, it is automatically true; if A is true, it is true only when B is true. When someone says "if A then B", and A is false, do you think he lied? for example, is the statement

"if sky is green, then 2+2=5"

true?

The usual convention in mathematics is that **it is true: any statement beginning with the words "if A then.. " is taken to be true in the case when A is false**.

The truth table for the operation IF:

A	B	$A \implies B$
T	T	T
T	F	F
F	T	T
F	F	T

7. Check whether $A \implies B$ and $B \implies A$ are equivalent, by writing the truth table for each of them.

8. On the island of Knights and Knaves, a traveler meets two inhabitants: Carl and Bill. Bill says: "Carl is a Knave". Carl says: "If Bill is a Knight, then I am a Knight, too."

Make a truth table and determine whether each of them is a Knight or a Knave?

Truth tables provide the easiest way to prove complicated logical rules: if we want to prove that two formulas are equivalent (i.e., always give the same answer), make a truth table for each of them, and if the tables coincide, they are equivalent. For example,

A	B	NOT A	NOT B	A AND B	NOT (A AND B)	(NOT A) OR(NOT B)
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

This table proves that NOT(A AND B) is the same as (NOT A) OR(NOT B).

9. Check that $A \implies B$ is equivalent to (NOT A) OR B by making a truth table for each of them (thus, "if you do not clean up your room, you will be punished" and "you clean up your room, or you will be punished" are the same).

10. Write the truth table for each of the following formulas. Are they equivalent (i.e., do they always give the same value)? (hint: there will be eight rows in the table)

(a) (A OR B) AND(A OR C)

(b) A OR(B AND C).

***11.** (The starred problems are those which are more difficult and thus are optional.) Define a new logical operation, XOR (exclusive or) as follows: $A \text{ XOR } B$ is true if exactly one of A, B is true, and false otherwise.

(a) Write the truth table for $A \text{ XOR } B$.

(b) Can you express XOR using only AND, OR, and NOT (that is, write a formula equivalent to $A \text{ XOR } B$ using only AND, OR, and NOT)?

***12.** The integers from 1 to 18 are written on the board in a row. Can you insert plus and minus signs between them in such a way as to get an expression that is equal to 0?