

ADVANCED MATH PROBLEM SOLVING CLUB  
SOME CLASSIC PUZZLES

APRIL 18, 2021

The puzzles given below were taken from a wonderful book “*Mathematical Puzzles: A Connoisseur’s Collection*” by Peter Winkler. However, he did not invent them: they are part of mathematical folklore which have been told and retold many times.

They are not united by a common theme, but I thought you might enjoy them anyway!

PROBLEMS

1. You are given two fuses (lengths of string treated with a special chemical making them burn steady). Each of them will burn for exactly 1 minute, but not uniformly along its length. Can you use them to measure 45 seconds?
2. A number of soldiers are stationed in a field in such a way all pairwise distances between them are distinct. Each soldier is told to keep an eye on the nearest neighbor.  
Show that if the number of soldiers is odd, then there is at least one soldier who is not watched.
3. After the revolution, each of 66 citizens of a certain country, including the king, is given the salary of one dollar. The king can no longer vote, but he does retain power to suggest changes — namely, the redistribution of salaries. Each person’s salary must be a whole number of dollars, and the sum of all salaries has to stay unchanged (66).  
Each suggestion is voted on, and is implemented only if there are more votes for than against. Each voter is selfish and will vote “yes” if his salary is increased, “no” if it will be decreased, and will not bother to vote if it is unchanged.  
Can the king increase his own salary? if so, what is the maximal salary he can get?
4. Alice, Bob, and Carol arrange a three-way duel. Alice is a poor shot, hitting her target only  $1/3$  of the time on average. Bob is better, hitting his target  $2/3$  of the time. Carol is a sure shot.  
They take turn shooting, first Alice (as she is the poorest shot), then Bob, then Carol (each can choose his or her target for each shot) and so on, until only one is left.  
What is Alice’s best course of action?
5. On a circular road there is a number of fuel depots. Together, they have just enough fuel to complete one loop of the road.  
Is it true that then, if you have a car with no gas, you can choose a starting position on this road so that you can make it all the way around?