

## MATH BATTLE!

JAN 10, 2021

1. A settlement consists of 9 blocks, forming a  $3 \times 3$  square, each block itself a square with side  $a$ . To deliver mail, the postman has to walk each of the streets (including the streets forming the outer boundary) at least once. What is the smallest distance he has to walk (starting from one of the corners)?
2. Rhino's skin has vertical and horizontal folds on its sides, 17 in total. If Rhino rubs one of its sides against a tree, then either two horizontal or two vertical folds on this side disappear, but two folds, one horizontal and one vertical, appear on the other side. (If there are less than two horizontal folds and less than two vertical folds, then nothing happens.)

Can it happen that the numbers of vertical and horizontal folds on each of Rhino's sides have been interchanged after Rhino scratched its sides several times? (e.g., if on his left side there were 3 horizontal and 4 vertical folds, then it would become 3 vertical and 4 horizontal, and similarly for the other side.)

3. Let  $n = 10^{20} - 2^{20}$ . Without using a calculator, find how many 2's will there be in the prime factorization of  $n$ ?
4. Black and White Stickers:

Ten students are in a classroom, wanting to leave, and the teacher decides to dismiss students according to their success in the following game: the teacher will put a black or a white sticker on each student's forehead (one per student, the color randomly chosen), so that the students can see each others' stickers but not their own. The teacher will then ask the students, one by one, to guess the color of their sticker. If they get it right, they may leave; if not, they must stay for the next class (which is very boring). During the game, starting from when the teacher places the first sticker, the students may not communicate at all, except that they may hear each others' guesses. The students are given five minutes to talk to each other before the game begins. They want to devise a strategy to save as many of them as possible. What's the best strategy? (Note: if anyone cheats, all students must stay for the next class.)