This part is just for fun

geometric series

$$(1 + r + r^2 + r^3 + \cdots + r^N)(1-r) = 1 - r^{N+1}$$

$$\sum_{i=0}^{N} r^i = \frac{1 - r^{N+1}}{1-r} \qquad N \to \infty \quad \sum_{i=0}^{\infty} r^i = \frac{1}{1-r}$$

$$\frac{1}{1-p^{-s}} = \sum_{n=0}^{\infty} p^{-ns} = \left(1 + \frac{1}{p^s} + \frac{1}{p^{2s}} + \frac{1}{p^{3s}} + \cdots\right.$$

$$\prod_p \frac{1}{1-p^{-s}} = \left(1 + \frac{1}{2^s} + \frac{1}{2^{2s}} + \frac{1}{2^{3s}} + \cdots\right) \cdot \left(1 + \frac{1}{3^s} + \frac{1}{3^{2s}} + \frac{1}{3^{3s}} + \cdots\right)\left(1 + \frac{1}{5^s} + \frac{1}{5^{2s}}\right.$$

$$\prod_p \frac{1}{1-p^{-s}} = \sum_{n=1}^{\infty} \frac{1}{n^s} = \zeta(s) \qquad \frac{1}{2^{3s}3^{2s}5} = \frac{1}{(2^3 \cdot 3^2 \cdot 5)^s}$$

Euler summation formula

definition of the zeta function

$$\zeta(2) = \frac{\pi^2}{6} \qquad \zeta(4) = \frac{\pi^4}{90}$$

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end of fun section

look up Riemann zeta function in YouTube.com

Numberphile

Reminder

An integer m can be written as

$$m = a \cdot x + b \cdot y$$

if and only if m is a multiple of $\gcd(a, b)$

$a = 18$
$= 2 \cdot 3^2$

$b = 33 = 3 \cdot 11$

$18x + 33y = 99$

$6\underline{x} + 11\underline{y} = 33$

$\gcd(11, 6)$

$\gcd(6, 5)$

$5 = 11 - 6$

$1 = 6 - 5 = 6 - 11 + 6 = 2 6 - 11$

$1 = 2 \cdot 6 - 1 \cdot 11$

$33 = 66 \cdot 6 - 33 \cdot 11$

Congruences

$$n_1 = q_1 \cdot m + r_1 \qquad r_1 < m$$

$$n_2 = q_2 \cdot m + r_2 \qquad r_2 < m$$

$$n_1 + n_2 = (q_1 + q_2) \cdot m + \underline{r_1 + r_2}$$

$$n_1 \cdot n_2 = (q_1 \cdot q_2 \cdot m + q_1 \cdot r_2 + q_2 \cdot r_1) \cdot m + r_1 \cdot r_2$$

$$a \equiv b \mod m \qquad 2 \equiv 9 \equiv 23 \equiv -5 \equiv -12 \mod 7$$

$$-12 = q \cdot 7 + r = -2 \cdot 7 + 2$$

$a \equiv a' \mod m$
$b \equiv b' \mod m$

$a+b \equiv a'+b' \mod m$
$a \cdot b \equiv a' \cdot b' \mod m$

$a \equiv 0 \mod m$ $\qquad m \mid a$

$5a \equiv 0 \mod m$ does not necessarily mean that $a \equiv 0 \mod m$

Problem 1

$\gcd(58, 38)$

$\gcd(38, 20)$

$\gcd(20, 18)$

$\gcd(18, 2)$

$\gcd(2, 0) = 2$

$58x + 38y = 4$

$29x + 19y = 2$

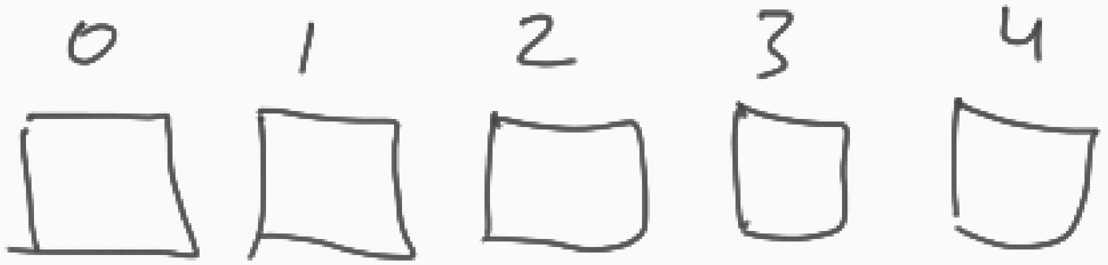
2 a)

for any a and m, the following sequence of remainders mod m

$a \bmod m$, $a^2 \bmod m$, $a^3 \bmod m$ ...

proof by contradiction

$a, a^2, \dots a^{m+1} \bmod m$, $m+1$ remainders



N pigeons holes   mod 5

N+1 pigeons.

b) $5^{1000} \equiv 1 \mod 12$

$5 \equiv 5 \mod 12$

$5^2 \equiv 1 \mod 12$

3.) $7^{2012} \equiv 1 \mod 10$ $\quad 11043 \equiv 3 \mod 10$

$7 \equiv 7 \mod 10$

$7^2 \equiv 49 \equiv 9 \mod 10$

$7^3 \equiv 9 \cdot 7 \equiv 3 \mod 10$

$7^4 \equiv 3 \cdot 7 \equiv 1 \mod 10$

$7^5 \equiv 7$

what is the last digit of

$$7^{7^{7}} = (7^7)^7 \equiv 3^7 \equiv 3^3 \equiv 7 \bmod 10$$

$7 \equiv 7 \bmod 10$

$7^2 \equiv 9 \bmod 10$

$7^3 \equiv 3 \bmod 10$

$7^4 \equiv 1 \bmod 10$

$$7^7 \equiv 7^3 \equiv 3 \bmod 10$$

$3 \equiv 3 \bmod 10$

$9 \equiv 9 \bmod 10$

$3^3 \equiv 7 \bmod 10$

$3^4 \equiv 1 \bmod 10$