

MATH 6 HANDOUT 6: SETS

SETS

Describing Sets. By word *set*, we mean any collection of objects: numbers, letters, ... Most of the sets we will consider will consist either of numbers or points in the plane. Objects of the set are usually referred to as *elements* of this set.

Sets are usually described in one of two ways:

- By explicitly listing all elements of the set. In this case, curly brackets are used, e.g. $\{1, 2, 3\}$.
- By giving some conditions, e.g. “set of all numbers satisfying equation $x^2 > 2$ ”. In this case, the following notation is used: $\{x \mid \dots\}$, where dots stand for some condition (equation, inequality, ...) involving x , denotes the set of all x satisfying this condition. For example, $\{x \mid x^2 > 2\}$ means “set of all x such that $x^2 > 2$ ”.

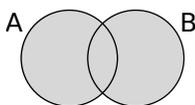
Members of sets. Sometimes we might have to say whether the element belongs to the set or not. In this case the following notation is used:

- $x \in A$ means “ x is in A ”, or “ x is an element of A ”
- $x \notin A$ means “ x is not in A ”

Set Operations. There are several operations that can be used to get new sets out of the old ones:

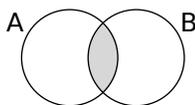
- $A \cup B$: *union* of A and B . It consists of all elements which are in either A or B (or both):

$$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}.$$



- $A \cap B$: *intersection* of A and B . It consists of all elements which are in both A and B :

$$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}.$$



- \bar{A} : *complement* of A , i.e. the set of all elements which are not in A : $\bar{A} = \{x \mid x \notin A\}$.

Intervals. The following notations are used when we talk about intervals on the number line. Intervals may have end points included or excluded: $[$ and $]$ represent that the end point is included, while $($ and $)$ indicate that the end point is excluded.

- $[a, b] = \{x \mid a \leq x \leq b\}$ is the interval from a to b (including endpoints),
- $(a, b) = \{x \mid a < x < b\}$ is the interval from a to b (**not** including endpoints),
- $[a, \infty) = \{x \mid a \leq x\}$ is the half-line from a to infinity (including a),
- $(a, \infty) = \{x \mid a < x\}$ is the half-line from a to infinity (**not** including a)

HOMEWORK

1. Consider the operation **NOR** which is just the opposite of **OR**: it returns **T** only if both **A** and **B** are **F**. Using only the component **NOR**, see if you can create circuits equivalent to **AND**, **OR**, and **NOT** as you did in the previous problem.
2. Using only **AND**, **NOT**, and **OR**, produce a three-input **AND** circuit, i.e., the output is **F** unless all three inputs are a **T**. (You do not have to use all three circuit elements.)
3. Using only **AND**, **NOT**, and **OR**, produce a three-input **OR** circuit, i.e., the output is **T** if any of the inputs is **T**.
4. If **Al** comes to a party, **Betsy** will not come. **Al** never comes to a party where **Charley** comes. And either **Betsy** or **Charley** (or both) will certainly come to the party.
Based on all of this, can you explain why it is impossible that **Al** comes to the party?
5. Let
 - A =set of all people who know French
 - B =set of all people who know German
 - C =set of all people who know RussianDescribe in words the following sets:
(a) $A \cap B$ (b) $A \cup (B \cap C)$ (c) $(A \cap B) \cup (A \cap C)$ (d) $C \cap \bar{A}$.
6. Let us take the usual deck of cards. As you know, there are 4 suits, hearts, diamonds, spades and clubs, 13 cards in each suit.
Denote:
 - H =set of all hearts cards
 - Q =set of all queens
 - R =set of all red cardsDescribe by formulas (such as $H \cap Q$) the following sets:
 - all red queens
 - all black cards
 - all cards that are either hearts or a queen
 - all cards other than red queensHow many cards are there in each set?
7. In a class of 25 students, 10 students know French, 5 students know Russian, and 12 know neither. How many students know both Russian and French?
8. Draw the following sets on the number line:
 - (a) Set of all numbers x satisfying $x \leq 2$ and $x \geq -5$;
 - (b) Set of all numbers x satisfying $x \leq 2$ or $x \geq -5$
 - (c) Set of all numbers x satisfying $x \leq -5$ or $x \geq 2$
9. For each of the sets below, draw it on the number line and then describe its complement:
 - (a) $[0, 2]$ (b) $(-\infty, 1] \cup [3, \infty)$ (c) $(0, 5) \cup (2, \infty)$