MATH 6: SPRING MATH BATTLE

COORDINATE GEOMETRY

MATH BATTLE

1. (a) If the vertices of parallelogram $ABCD$ are $A(-2,3)$, $B(1,2)$, $C(3,5)$ and $D(0,k)$, find k.
   (b) The points $(2,-3)$, $(2,3)$ and $(k,0)$ lie on the same straight line. What is the value of k?
   (c) If $(r,3)$ is in the solution set of $2x + y = 7$, then what is the value of r?

2. Let O be the origin $(0,0)$ of the xy-plane, and let C_1, C_2 be two circles whose equations are $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.
   Now, given any point A, the ray $\overrightarrow{OA}$ intersects both C_1 and C_2 - let these points be called A_1 and A_2 (they will depend on the choice of A).
   Write down the equation for the locus of points A in the xy-plane such that A is the midpoint of $\overline{A_1 A_2}$.

3. (a) Write the equation of a line that is parallel to the line $2y = 6x - 1$ and has a y-intercept of -3.
   (b) Find the slope of the line passing through $(7,-2)$ and $(-4,-1)$.
   (c) Find k so that the slope of the line passing through the points $(2,k)$ and $(6,8)$ is 1.

4. (a) Given the lines $y = 2x + 1$, $y = 1$, $x = 2$, find the coordinates of the three vertices formed by their intersections, then find the area of the triangle.
   (b) Prove that it is possible to put this triangle together with three congruent copies of itself to form a rhombus. Write down what the coordinates of the rhombus would be.

5. Find the number of arrangements of the letters in the word BOOKKEEPER.

6. How many arrangements of the letters in the word LOOT include a double-O?

7. What is the sum $\dfrac{1}{3} + \dfrac{1}{3^2} + \dfrac{1}{3^3} + \ldots$

8. Given the arithmetic sequence: 4,11,18,25,...
   What is the n-th term?