MATH 6
HOMEWORK 6: SETS CONTINUED

New material introduced today:

We say that set A is a *subset* of B (notation: $A \subseteq B$) if every element of A is also an element of B: $x \in A \Rightarrow x \in B$. Note that A can be equal to B. (Sometimes the notation $A \subset B$ is used, in which case it may or may not include equality, but we will use $\subseteq$ for both cases.)

Logically, we write the definition of $A \subseteq B$ as, for all x, $x \in A \implies x \in B$.

Additionally, it is useful to note that $(A \subseteq B)$ AND $(B \subseteq A)$ means that $A = B$.

1. Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$. Is it true that $A \subseteq B$?
Can you find an element of B that is not an element of A?

2. Let
A=set of all people who know French
B=set of all people who know German
C=set of all people who know Russian
Describe in words the following sets:
(a) $A \cap B$ (b) $A \cup (B \cap C)$ (c) $(A \cap B) \cup (A \cap C)$ (d) $C \cup \overline{A}$.

3. Let us take the usual deck of cards. As you know, there are 4 suits, hearts, diamonds, spades and clubs, 13 cards in each suit.
Denote:
H=set of all hearts cards
Q=set of all queens
R=set of all red cards
Describe by formulas (such as $H \cap Q$) the following sets:
all red queens
all black cards
all cards that are hearts or a queen
all cards other than red queens
How many cards are there in each set?

4. In a class of 25 students, 10 students know French, 5 students know Russian, and 12 know neither. How many students know both Russian and French?

***5.** A local frog named Filo running for parliament tells the public, at the famous amphibian debate convention, that Filo will speak for all frogs who do not speak for themselves, and Filo will speak for no one else. Can you logically deduce if Filo will speak for Filo's own self?

6. Consider the following sets:
$\mathbb{Z}$ — all whole numbers (positive and negative)
$\mathbb{N}$ — all positive whole numbers
$\mathbb{R}$ — all numbers
$\mathbb{Q}$ — all rational numbers (i.e., those that can be written as a fraction)
Order them from smallest to largest, so that each set is a subset of the next one.

7. List all subsets of the set $S = \{1, 2, 3\}$ (do not forget the empty set $\varnothing$ and S itself). Can you guess the general rule: if set S has n elements, how many subsets does it have?

***8.** (a) Let $S = \{1, 2, 3, 4, 5\}$ and let $A \subseteq S$ such that A is a subset of exactly 4 subsets of S, including S itself and A itself. Can you determine how many elements are in A?

(b) Let $T_n = \{1, 2, 3, ..., n\}$, and similarly $T_k = \{1, 2, 3, ..., k\}$, with $k < n$. How many subsets of T_n is T_k a subset of? Don't forget to count T_n and T_k themselves.

***9.** Let A be the set of all ordered pairs of numbers (x, y) such that $0 < x < 1$, and B be the set of all ordered pairs of numbers (x, y) such that $0 < y < 1$.

(a) Is $(0.5, 1)$ in A? Is it in B?

(b) Determine whether the following points are in $A \cup B$: $(0.1, 0.2)$, $(0.1, 2.5)$, $(1.1, 1.1)$, $(-2, 0.5)$.

(c) Prove for any two numbers r, s the following statement: $(r, s) \in (A \cup B) \iff (s, r) \in (A \cup B)$.

(d) Prove $(r, s) \in (A \cup B) \iff (1 - s, 1 - r) \in (A \cup B)$.

10. (AMC) A box contains five cards, numbered 1, 2, 3, 4, and 5. Three cards are selected randomly without replacement from the box. What is the probability that 4 is the largest value selected?