MATH 6
HOMEWORK 5: SETS

Sets

Sets are wonderful things! They represent collections of objects (where those objects could be anything - numbers, letters, etc.). We often use letters to represent individual sets in the same way we used letters to represent logic statements; this lets us work with operations on sets, and to an extent think of them as objects in their own right.

Any object that's inside a set is known as an *element* of the set. For this we use the symbol $\in$, for example we say $x \in A$ means that item x is in set A or is an element of A. Similarly we use $\notin$ to mean "is not an element of", so $x \notin A$ means x is not an element of A.

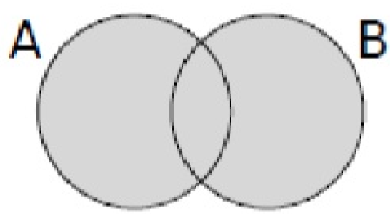
Sets are usually described in one of two ways:

- By explicitly listing all elements of the set. In this case, curly brackets are used, e.g. $\{1, 2, 3\}$. Here we have $1 \in \{1, 2, 3\}$, $2 \in \{1, 2, 3\}$, $3 \in \{1, 2, 3\}$.
- By giving some conditions, e.g. "set of all numbers satisfying equation $x^2 > 2$". In this case, the following notation is used: $\{x \mid \dots\}$, where dots stand for some condition (equation, inequality, ...) involving x, denotes the set of all x satisfying this condition. For example, $\{x \mid x^2 > 2\}$ means "set of all x such that $x^2 > 2$". Such a set may have infinitely many numbers in it.

The empty set: $\varnothing$ is the empty set, or set which contains no elements. It is sometimes useful for the same reasons it is useful to have a notation for number 0.

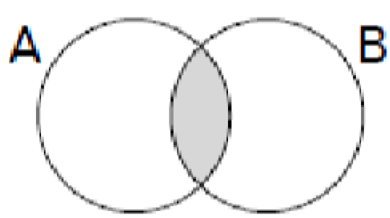
Union: $A \cup B$ is the union of A and B. It is the set of all elements which are in either A or B (or both):

$$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}.$$



Intersection: $A \cap B$ is the intersection of A and B. It is the set of all elements which are in both A and B:

$$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}.$$



Complement: $\overline{A}$ is complement of A. It is the set of all elements which are not in A: $\overline{A} = \{x \mid x \notin A\}$.

Count: $|A|$ number of elements in a set A. If A has finitely many elements, $|A|$ will be a number; otherwise, we say $|A|$ is infinite.

1. If Ariel is cast a movie, Belle will refuse to be in the cast. Ariel is never cast in a movie where Jasmine is in the cast. And either Belle or Jasmine (or both) will certainly star in Disney's upcoming movie.

Based on all of this, can you explain why it is impossible that Ariel be in the cast of Disney's upcoming movie?

2. Let $A = [1, 3] = \{x \mid 1 \le x \le 3\}$, $B = \{x \mid x \ge 2\}$, $C = \{x \mid x \le 1.5\}$. Describe $\overline{A}$, $\overline{B}$, $\overline{C}$, $A \cap B$, $A \cap C$, $A \cap (B \cup C)$, $A \cap (B \cap C)$.

3. Draw the following sets on the number line:
(a) Set of all numbers x satisfying $x \le 2$ and $x \ge -5$;
(b) Set of all numbers x satisfying $x \le 2$ or $x \ge -5$
(c) Set of all numbers x satisfying $x \le -5$ or $x \ge 2$

4. Using Venn diagrams, explain why $(A \cap B) \cap C = A \cap (B \cap C)$. Deduce that the notation $A \cap B \cap C$ is unambiguous (indeed, this triple intersection refers to the set of every element that is in all of A, B, and C).

5. (a) Using Venn diagrams, explain why $\overline{A \cap B} = \overline{A} \cup \overline{B}$. Does it remind you of one of the logic laws we had discussed before?
(b) Do the same for formula $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

6. In this problem, we denote by $|A|$ the number of elements in a finite set A.
(a) Show that for two sets A, B, we have $|A \cup B| = |A| + |B| - |A \cap B|$.
*(b) Can you come up with a similar rule for three sets? That is, write a formula for $|A \cup B \cup C|$ which uses $|A|, |B|, |C|$, $|A \cap B|$, $|A \cap C|$, $|B \cap C|$, $|A \cap B \cap C|$.

7. Let A be the set of even integers and B be the set of multiples of 3.
(a) Is it possible to find a set C such that $(A \cup B) \cup C = A$?
(b) Is it possible to find a set C such that $(A \cup B) \cap C = A$?

8. (a) Prove (by truth table or otherwise) that B OR A OR B is not equivalent to A.
(b) Prove (by truth table or otherwise) that B XOR A XOR B is equivalent to A.

9. (AMC) For the game show Who Wants To Be A Millionaire?, the dollar values of each question are shown in the following table (where K = 1000).

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Value	100	200	300	500	1K	2K	4K	8K	16K	32K	64K	125K	250K	500K	1000K

Between which two questions is the percent increase of the value the smallest?

10. (AMC) Order the following numbers from smallest to largest: 10^8, 5^{12}, 2^{24}.

11. (AMC) Which of the following has the largest shaded area?