MATH 6
HOMEWORK 1: KNIGHTS AND KNAVES

WELCOME TO A NEW SEMESTER AT SCHOOLNOVA!!

PROBLEMS

In this homework assignment (and in all other assignments in this class), many problems are non-trivial and require some thought. Try to start early. You are not expected to be able to solve all of the problems, so do not be discouraged if you can't solve some of them. The solutions are to be written on separate sheets of paper (as neatly as possible), with your name at the top, and handed back to me by the next class. Please make sure that you write not just the answer but also the solution, i.e. your reasoning showing how you arrived to this answer. Ideally, your solution should be such that someone who doesn't know how to solve this problem can read it and follow your arguments. All homeworks will be posted on the schoolnova.org site.

Many of the questions of this assignment refer to the famous (among logic puzzle fans) island of Knights and Knaves. On this island, there are two kinds of people: Knights, who always tell the truth, and Knaves, who always lie. Unfortunately, there is no easy way of knowing whether a person you meet is a knight or a knave...

Copyright notice: most of these problems come from books of Raymond Smullyan. If you liked them, get his books in the library and you will find there many more puzzles of the same sort.

1. Find the greatest common divisor and least common multiple of 132 and 90.

2. Solve the following equation: $5 - 2(1 - x) = 9$.

3. (This question should be answered without using a calculator.) The maximal distance from Sun to Pluto is 7,375,927,931 km. Speed of light is about 300,000 km/sec. How long does it take for Sun's light to reach Pluto? (You do not need to give a precise answer — an approximate one like "about 2 minutes" would be fine.)

4. You toss a coin 2 times. Find the probability that
   (a) you get a head followed by a tail
   (b) you get a tail followed by a head
   (c) you get a tail and a head, it doesn't matter what order
   (d) you get two heads
   (e) that you get the same result in both tosess (i.e., either two heads or two tails.)
   (Hint: consider all 4 cases HH,TT,HT,TH, where H-head, T-tail)

5. You toss a coin 3 times. Find the probability that
   (a) you get a head followed by 2 tails
   (b) you get a head and 2 tails it doesn't matter in what order
   (c) you get three heads
   (d) that you get the same result in all three tosess (i.e., either three heads or three tails.)
   (Hint: Consider all 8 cases HHH,THH,HTH,HHT,THT,TTH,HTT,TTT)

6. On the island of knights and knaves, you meet two inhabitants: Zoey and Mel. Zoey tells you that Mel is a knave. Mel says, "Neither Zoey nor I are knaves." So who is a knight and who is a knave?

7. On the island of knights and knaves, you meet two inhabitants: Sue and Zippy. Sue says that Zippy is a knave. Zippy says, "I and Sue are knights." So who is a knight and who is a knave?

8. On the island of knights and knaves, you meet two inhabitants: Bart and Ted. Bart claims, "I and Ted are both knights or both knaves." Ted tells you, "Bart would tell you that I am a knave." So who is a knight and who is a knave?

9. A traveler to the island of Knights and Knaves meets a group of five people (call them A, B, C, D, E).
    A says: "exactly one of us is a Knight"
    B says: "exactly two of us are Knights"
    C says: "exactly three of us are Knights"
    D says: "exactly four of us are Knights"
    E says: "all five of us are Knights"
    Can you find out which of them are Knights?