MATH 6: SPRING MATH BATTLE

APRIL 19, 2020

(homework on the next page)

Math Battle

1. Prove that a rectangle whose four sides are all tangent to the same circle must in fact be a square.

2. (a) Given the lines $y = 2x+1$, $y = 1$, $x = 2$, find the coordinates of the three vertices formed by their intersections, then find the area of the triangle.

(b) Prove that it is possible to put this triangle together with three congruent copies of itself to form a rhombus. Write down what the coordinates of the rhombus would be.

(c) Write down the coordinates of four points that form a quadrilateral whose diagonals are perpendicular but the quadrilateral is not a rhombus. (Your example does not have to be related to the rhombus of part b - you can place your points wherever in the xy-plane you feel is most useful.)

3. Let $\triangle ABC$ be a right triangle with legs $\overline{AB}$ of length 5 and $\overline{BC}$ of length 12, and hypotenuse $\overline{AC}$ of length 13. Let D be on $\overline{AC}$ such that $\overline{BD} \perp \overline{AC}$.
Now let l be the line parallel to $\overline{AB}$ through C, and m be the line parallel to $\overline{BC}$ through A. Find the distance between the intersection point of $\overleftrightarrow{BD}$ with l and the intersection point of $\overleftrightarrow{BD}$ with m. (Express your answer as a fraction.)

4. Prove that $(A \iff (B \text{ AND } C)) \text{ AND} (B \iff (C \text{ AND } A)) \text{ AND} (C \iff (A \text{ AND } B))$ is equivalent to $(A \iff (B \text{ OR } C)) \text{ AND} (B \iff (C \text{ OR } A)) \text{ AND} (C \iff (A \text{ OR } B))$

5. I have several identical pieces of candy that I wish to distribute among my three children. Assuming I am unbothered by shameless favoritism, yet somehow still hold to the moral principle that no child may receive more than five pieces of candy, prove that the number of ways to distribute five pieces of candy among my three children is equal to the number of ways to distribute ten pieces of candy among my three children.

6. A hiker climbs a mountain, starts at 9am and reaches the summit at 12pm. The next day, he returns and starts again at 9am and reaches the base of the mountain at 12pm. Show that there exists a point on his trip where he stood at exactly the same time each day.