

MATH 6
HOMEWORK2: SOLVING EQUATIONS AND INEQUALITIES

SOLVING INEQUALITIES INVOLVING NEGATIVE NUMBERS

Today we discussed inequalities and their solutions, discussing how one solves inequalities involving negative numbers. The rule here is

$$a < b \iff (-a) > (-b)$$

(note that the sign of inequality is reversed!). For example, $3 < 5$, but $-3 > -5$.

More generally, if we multiply or divide both sides of an inequality by a negative number, we need to change the sign of inequality, replacing $<$ by $>$ and vice versa. For example, to solve $-3x > -6$, we divide both sides by -3 and change $>$ to $<$, giving $x < 2$.

PRODUCTS

The following rules are frequently used when dealing with equations or inequalities where the left-hand side is a product of two factors:

- A product of two numbers is zero if and only if one of them is zero:

$$ab = 0 \iff (a = 0) \text{ OR } (b = 0)$$

- A product of two numbers is positive if and only if both numbers are positive or both numbers are negative:

$$ab > 0 \iff (a > 0 \text{ AND } b > 0) \text{ OR } (a < 0 \text{ AND } b < 0)$$

- A product of two numbers is negative if and only if one of numbers is positive and the other one is negative:

$$ab < 0 \iff (a > 0 \text{ AND } b < 0) \text{ OR } (a < 0 \text{ AND } b > 0)$$

HOMEWORK

1. Hello! Recall that negative numbers have some unique properties that require you to be careful when making calculations. If you ever feel confused, don't hesitate to draw a brief number line to make visual comparisons if it helps. Anyways, please simplify the following expressions into single numbers.

$$\begin{array}{lll} -3 - (-2) & -3 - 3 & ((-4) \times 6) - 7 \\ (-6) \div (-2) + 3 & (-2) \div (-3) & (-4) \times (-7) \div (+9) \\ (-4) - (-3) + (-5) & (-6) + (-2) \div (-3) & \end{array}$$

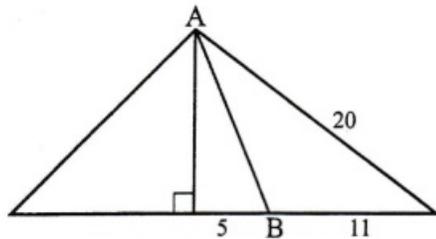
2. Solve the following inequalities

$$\begin{array}{lll} \text{(a)} \quad -x < 2 & \text{(b)} \quad 2 - 3x > 5 & \text{(c)} \quad 3x + 1 < 5x + 7 \\ \text{(d)} \quad 1 + 5x < 3x & \text{(e)} \quad x - 1 < x - 7 & \end{array}$$

3. Solve the following equations and inequalities:
 (a) $(x - 1)(x - 2) = 0$ (b) $x(x + 1) < 0$ (c) $\frac{1}{x} > 2$ [Hint: multiply by x]
 (d) $x^2 - 4 = 0$ (e) $\frac{x}{x+1} > 1$ [Hint: consider separately two cases: $x + 1 > 0$ and $x + 1 < 0$]
4. In the figure below, each symbol stands for a number. The sum of numbers in each column or row is written next to the column or row — except for the second column, where the sum is not known. Can you find this missing sum?

⊙	☆	▲	▲	96
▲	⊙	⊙	⊙	92
☆	☆	☆	▲	140
⊙	▲	⊙	☆	108
108	?	108	96	

5. Old MacDonald raises sheep and chickens on his farm. His livestock has a total of 55 heads and 142 legs among them (not counting the farmer!). How many chickens and how many sheep does he have?
6. Mr. Sim jogs at 9 km per hour over a certain distance and walks back the same distance at 6 km per hour. What is his average speed? [Hint: the answer $(6 + 9)/2 = 7.5$ is wrong!].
7. Show that in a class of 40 students, there must be two students whose birthdays are less than 10 days apart.
8. Find AB (Mathcounts).



9. Three cubes are stacked as shown. If the cubes have edge 1, 2 and 3 what is AB? (Mathcounts)

