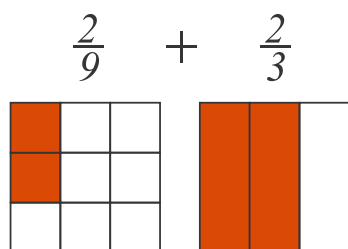


### Addition and subtraction of fractions with unlike denominators.

Can we easily compute  $\frac{2}{9} + \frac{2}{3}$ ? Let's now try to add  $\frac{2}{9}$  and  $\frac{2}{3}$ .



Are two fractions  $\frac{2}{9}$  and  $\frac{2}{3}$  similar objects?

$$\frac{2}{9} = \frac{1}{9} + \frac{1}{9}$$

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$$

How we can calculate?

$$\frac{2}{9} + \frac{2}{3} = \frac{1}{9} + \frac{1}{9} + \frac{1}{3} + \frac{1}{3}$$

- To be able to add two fractions we must rewrite them as fractions with **the same denominator**.
- The best choice for such common denominator is the **least common multiple (LCM)** of the denominators of original fractions.
- When you find the **factor** by which you need to multiply your denominator, remember to **multiply the numerator by the same number** so that your fraction remains the same
- For example,

$$\frac{3}{9} + \frac{2}{3} = \frac{3}{9} + \frac{2 \times 3}{3 \times 3} = \frac{3}{9} + \frac{6}{9} = 1$$

### Multiplication of a whole number by a fraction.

$$\frac{2}{3} \times 5 = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} \text{ (we add } \frac{2}{3} \text{ to itself 5 times)}$$

Of course we remember how to add fractions with the same denominator:

$$\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{2 + 2 + 2 + 2 + 2}{3}$$

$$\frac{2 + 2 + 2 + 2 + 2}{3} = \frac{2 \times 5}{3}$$

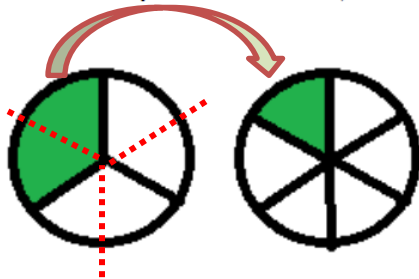
$$\frac{2}{3} \times 5 = \frac{2 \times 5}{3}$$

To multiply a fraction by a whole number, multiply the numerator by this number

$$\frac{a}{b} \times c = \frac{a}{b} \times \frac{c}{1} = \frac{a \times c}{b}$$

### Multiplication of a fraction by a fraction.

Analogously,  $\frac{1}{2} \times \frac{1}{3}$  means  $\frac{1}{2}$  of  $\frac{1}{3}$ . Now, half of  $\frac{1}{3}$  piece of a disk is  $\frac{1}{6}$  of a disk (look at the picture below).



Notice that we could have just multiplied the denominators of  $\frac{1}{2}$  and  $\frac{1}{3}$ .

To multiply fraction by a fraction, multiply the numerators to get the numerator for the answer, multiply denominators to get denominator for the answer.

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

### Reciprocal Fractions:

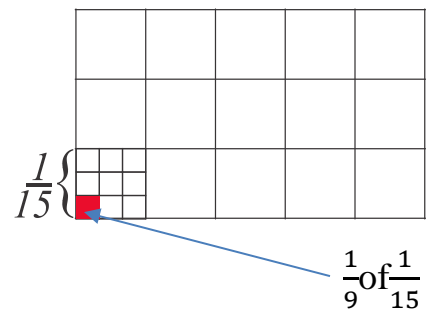
Two fractions are called reciprocal if their product is equal to 1.

$$\frac{3}{5} \times \frac{5}{3} = 1, \quad \frac{a}{b} \times \frac{b}{a} = 1$$

### Division of Fractions:

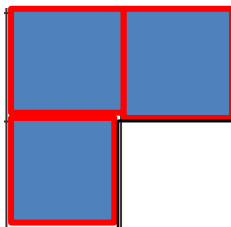
If I multiply two fractions (let say  $\frac{1}{15}$  by  $\frac{1}{9}$ ), I will find one ninth part of one fifteenth (or one fifteenth part of one ninth).

$$\frac{1}{15} \cdot \frac{1}{9} = \frac{1}{15} \div 9 = \frac{1}{15 \cdot 9} = \frac{1}{135}$$

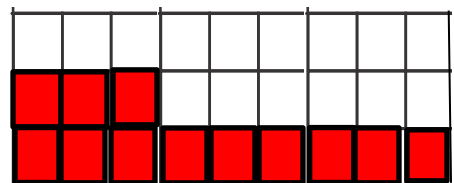


**To divide one fraction by another (if they have common denominator) you need to divide only their numerators**

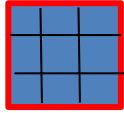
$$\frac{3}{4} : \frac{1}{4} = 3:1 = 3$$



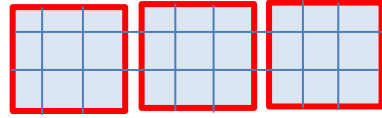
$$\frac{12}{27} : \frac{6}{27} = 12:6 = 2$$



$$1 : \frac{1}{9} = 9$$

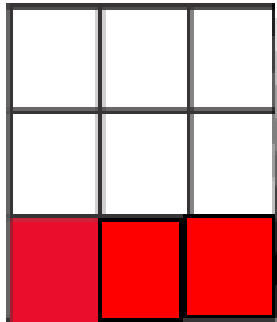


$$3 : \frac{1}{9} = 27$$



To divide a number (or a fraction) by a fraction we should multiply by its

reciprocal  $a : \frac{c}{d} = a \cdot \frac{d}{c}$



$$\frac{3}{9} : \frac{1}{3} = \frac{3}{9} : \frac{3}{9} = 1 = \frac{3}{9} \times \frac{9}{3}$$

$$\frac{3}{9} \times \frac{9}{18} = \frac{3 \times 9}{9 \times 18} = \frac{27}{162} = \frac{27 \div 27}{162 \div 27} = \frac{1}{6}$$

We brought both fractions to the same denominator “9·18”

Now we can just divide the numerators

When we do that, it turns out that we are multiplying the original fraction

$$\frac{3}{9} \text{ (dividend) by the reciprocal of the divisor } \frac{18}{9}$$