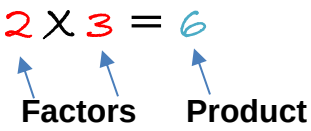
Factors

"Factors" are the numbers you multiply together to get another number:



Factorization

Factorization is breaking(decomposing) a number by its factors. It is the same as representing a number as a product of two or more other numbers, for example:

- $40 = 4 \times 10$, $36 = 6 \times 6$
- $a \times b + a \times c = a \times (b + c)$
- $7 \times 5 + 7 \times 3 = 7 \times (5 + 3)$

- A natural number that is greater than 1 and has no positive divisors other than 1 and itself is called a prime number.

- Even numbers are the numbers divisible by 2 (they have 2 as a divisor), so they can be factorized as 2 times something else.

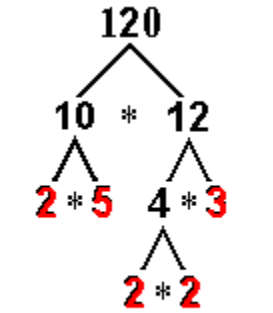
Prime numbers

Prime numbers are building blocks of all composite numbers. Prime number can only be divided by 1 or itself, so it cannot be factored any further. Positive integers other than 1 which are not prime are called composite numbers.

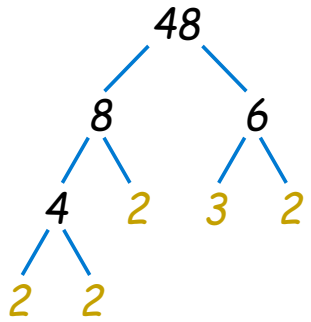
Prime <u>factorization</u> is finding **which prime numbers** multiply together to make the original number

There is only one (unique!) set of prime factors for any natural number.

Prime factorization process



2*2*2*3*5=120



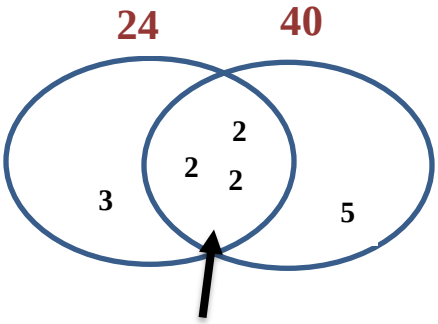
48 = 2 x 2 x 2 x 2 x 3

Greatest Common Factor (Divisor)

The largest number that divides two given integers without remainder

To find GCF of two integers 24 and 40:

1. Prime factorize 24=6x4=2x3x2x2 40 =4x10=2x2x5x2
2. Find pairs 24=6x4=2x3x2x2 40 =4x10=2x2x5x2
3. Multiply them 2x2x2=8
4. Both numbers are divisible by 8. GCF(24, 40)=8



Greatest Common Factor

Word Problem 1

For Halloween the Jonson family bought 168 mini chocolate bars and 180 gummy worms. What is the largest number of kids between whom the Jonson family can divide both kinds of candy evenly?

To solve this problem we have to find the Greatest Common Divisor (GCD), the largest number that can be a divisor for both 168 and 180.

Let's look at a set of all prime factors of 168 and 180.

For 168 = 2x2x2x3x7. Any of these numbers as well as any of their products can be a divisor of 168.

For 180= 2x2x3x3x5.

These two sets have common elements (factors). It means that both numbers are divisible by any of these common factors and by their products.

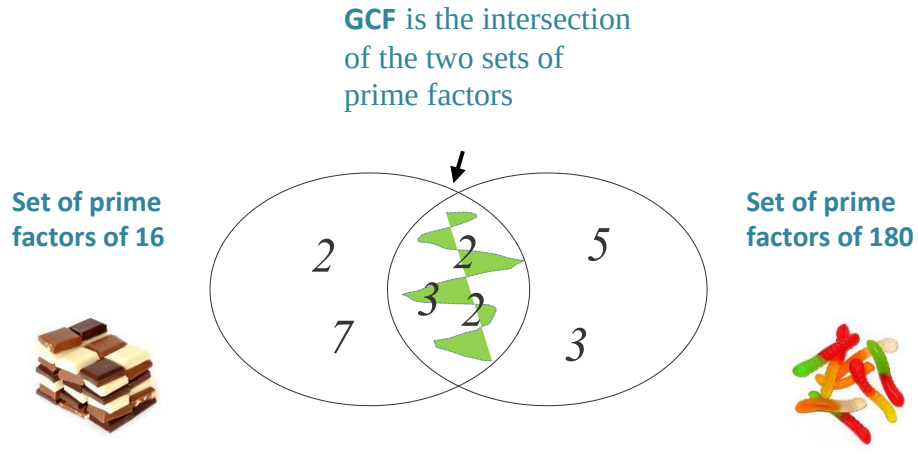
The greatest common factor (GCF) is the product of all common factors 2x2x3=12. So the candy can be divided evenly between 12 kids.

Each kid gets $168 \div 12 = 14$ chocolate bars

$180 \div 12 = 15$ gummy worms

Prime factors of* 168 *and* 180 *are $2 \times 2 \times 2 \times 3 \times 7 = 168$; $2 \times 2 \times 3 \times 3 \times 5 = 180$

GCF is the common set of prime factors. One the Venn Diagram you can visualize it as



Least Common Multiple (LCM)

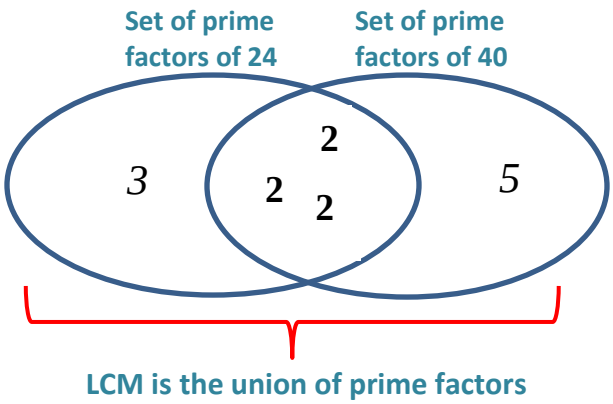
LCM is the smallest number that has given integers as factors. LCM is the smallest number that is a multiple of each of given integers.

To find LCM of two integers 24 and 40 you can find their multiples:

Multiples of 24	Multiples of 40
24x1=24	40x1=40
24x2=48	40x2=80
24x3=72	40x3=120
24x4=96	40x4=160
24x5=120	40x5=200

To find LCM you can also Prime factorize:

1. Prime factorize 24=6x4=2x3x2x2 40 =4x10=2x2x5x2
2. Multiply factors that represent each integer 2x2x2x3x5=120
3. LCM (24, 40) =120

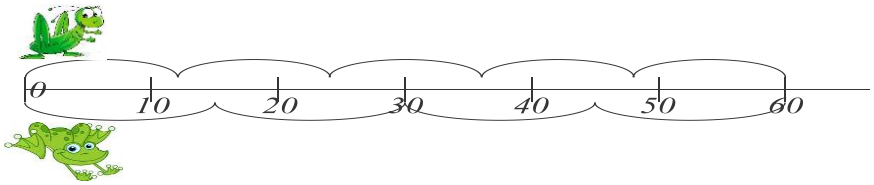


Word Problem 2

A grasshopper jumps the 12 cm distance each jump. A little frog jumps the 15 cm distance each jump. They start jumping from the point 0 and jump along a big ruler. What is the closest point (measure) on the ruler they both can land?

We should find the smallest number that is divisible by both 12 and 15 (i. e. LCM)

One method is to find multiples of 12 and multiples of 15

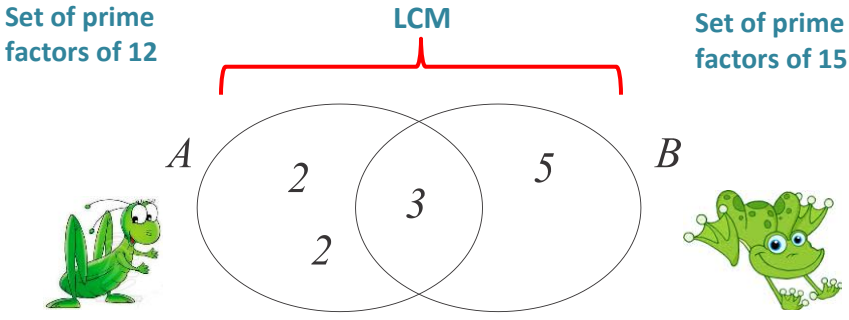


Multiples of 12	Multiples of 15
12x1=12	15x1=15
12x2=24	15x2=30
12x3=36	15x3=45
12x4=48	15x4=60
12x5=60	15x5=75
12x6=72	15x6=90

We also can do it using prime factorization of 12 and 15: $12 = 2 \times 2 \times 3$ $15 = 3 \times 5$

LCM: $2 \times 2 \times 3 \times 5 = 60$

The number which we are looking for must be a product of prime factors of 12 AND 15, so it should be a union of two sets – set of prime factors of 12 and 15.



$2 \times 2 \times 3 \times 5 = 60$

60 is the smallest number, which is divisible by 12 and 15, LCM.

We found that 60 is the smallest number, which is divisible by 12 and 15, LCM. So, both the frog and the grasshopped will be eventually landing on the 60 mark on our ruler.