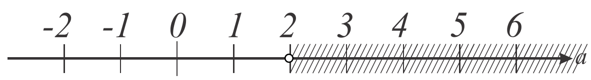
Math 4. Handout #23

Inequalities.

An equation is the problem of finding values of some variables, called *unknowns*, for which the specified equality is true. To find the value of the unknown variable you must solve the equation. There are another kind of mathematical statements – inequalities.

Which a can satisfy the statement: $a > 2$?

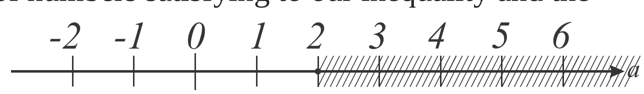


As we can see all a which lie on the right side of number 2 will satisfy the expression $a > 2$. What about number 2 itself? Number 2 does not satisfy our expression. How we can wright the answer for $a > 2$?

The best way to write the answer in terms of set theory: $a \in]2, \infty)$ ($a \in (2, \infty)$), or the answer is set of points of number line located on the right side of number 2.

- Now let's solve the inequality $a \geq 2$.

In this case number 2 itself also belongs to the set of numbers satisfying to our inequality and the answer will be $a \in [2, \infty)$

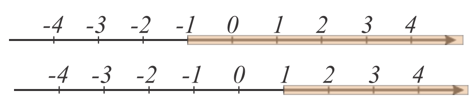


Rules of inequalities

We can add any number to both part of the inequality, the sign (< or >) will not change:

$x > -1$

$x + 2 > -1 + 2 \Rightarrow x + 2 > 1$

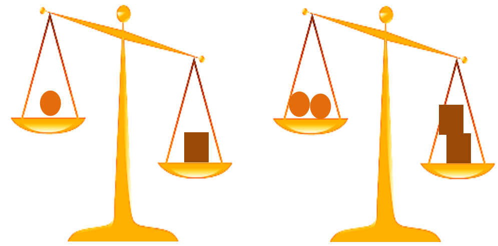


$y - 3 < 5$

$y - 3 + 3 < 5 + 3y < 8$

$y \in (-\infty, 8)$

Now let's try to multiply or divide both parts of the inequality by a positive number.



If $x > 3$, then $2x$ will be grater then 6.

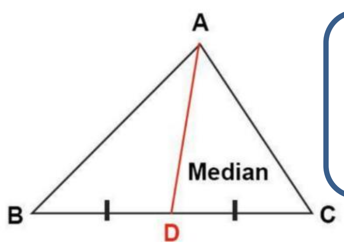
$x > 3$, $2x > 6$

How about multiplying by a negative number?

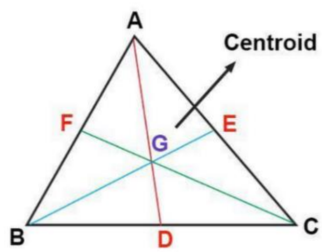
When you multiply or divide each side of an inequality by a negative number- you have to swap the inequality:

1<2   -1x1 > -1x2

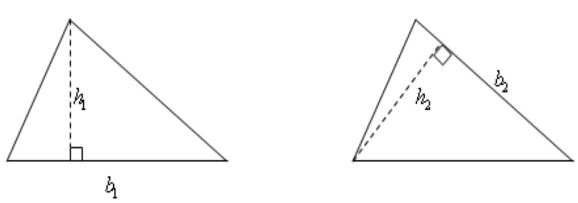
Geometry: Median, Height, Bisector



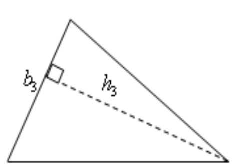
- A **median** of a triangle is a segment that connects any vertex of the triangle with the mid-point of its opposite side
- A **median** of a triangle divides the triangle into two equal areas

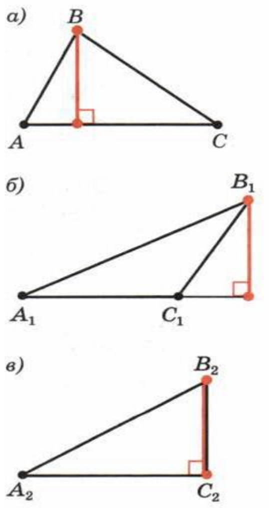


- The point where all three medians of a triangle intersect called 'centroid'

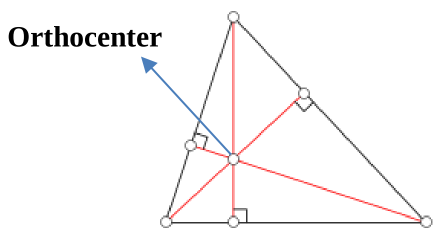


Altitude (or the height) of a triangle is a segment drawn from one vertex of a triangle perpendicular to the line which contains the opposite side.

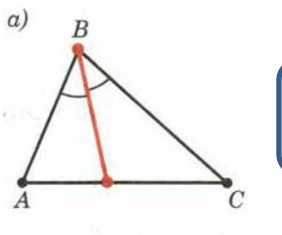




- Note that on the pictures on the left the altitude is not always falling inside the opposite side of the triangle – it may cross the line which contains that side outside the triangle!



- The point where all altitudes of a triangle intersect called “orthocenter”.

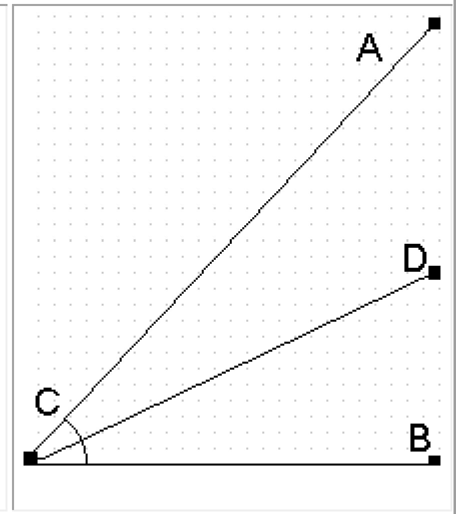


The Bisector of an angle is a ray whose end point is the vertex of the angle and which splits the angle onto two equal angles.

How to construct angles and various geometrical figures :

https://www.mathopenref.com/tocs/constructionstoc.html

In the diagram to the right, the ray CD is the bisector of the angle ACB if and only if the angles ACD and BCD have equal measures.



How to construct an angle bisector:

John D. "Math Open Reference" www.mathopenref.com

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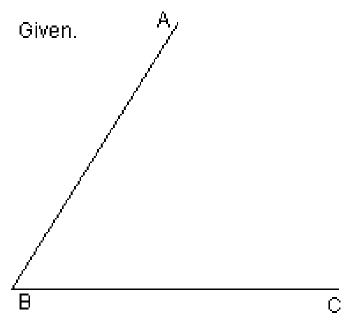
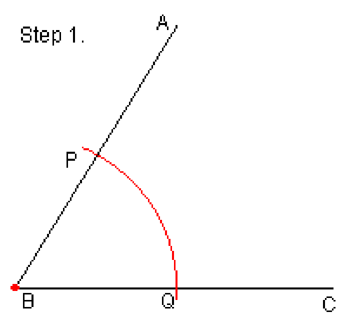
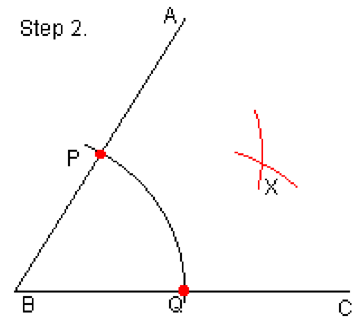
You are given an angle. Let’s call it ABC.



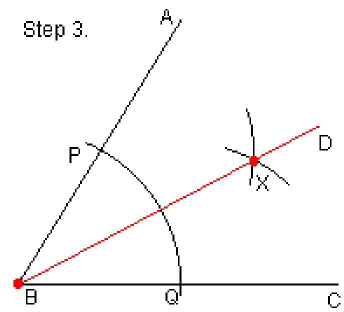
Step 1. Draw an arc that is centered at the vertex of the angle. This arc can have a radius of any length. However, the arc must intersect both sides of the angle. Let’s call these intersection points **P** and **Q.** This provides a point on each line that is an equal distance from the vertex of the angle.



Step 2. Draw two more arcs. The first arc must be centered on one of the two points **P** or **Q**. It can have any length radius. The second arc must be centered on whichever point (P or Q) you did NOT choose for the first arc. The radius for the second arc MUST be the same as the first arc. Make sure you make the arcs long enough so that these two arcs intersect in at least one point. Let's call this intersection point **X**. Every intersection point between these arcs (there can be at most 2) will lie on the angle bisector.



Step 3. Draw a line that contains both the vertex and **X**. Since the intersection points and the vertex all lie on the angle bisector, we know that the line which passes through these points **must** be the angle bisector.



A video tutorial and printable instructions on how to construct an altitude of a triangle:

https://www.mathopenref.com/constaltitude.html

https://www.mathopenref.com/printaltitude.html

A video tutorial and printable instructions on how to construct a median of a triangle:

https://www.mathopenref.com/constmedian.html

https://www.mathopenref.com/printmedian.html