



## REVIEW Proper and improper fractions. Mixed numbers

## Proper and improper fractions:

**An improper fraction** is a fraction in which the numerator (top number) is greater than or equal to the denominator (bottom number).

Fractions such as  $\frac{6}{5}$  or  $\frac{8}{5}$  are “improper”.

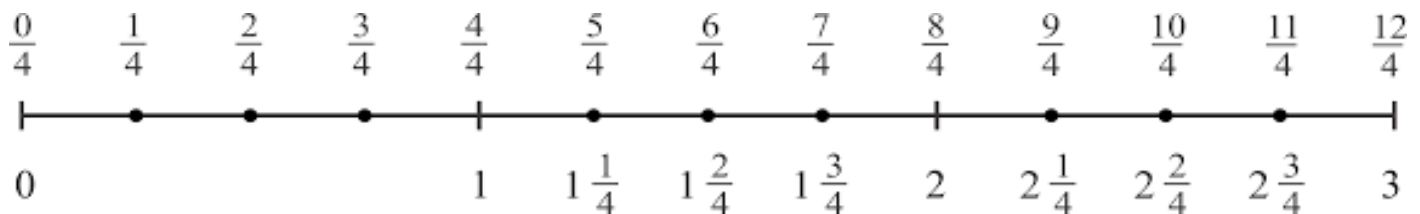
The improper fraction is just another way of writing a mixed number.

3. On the number line below, find an improper fraction which correspond to the following mixed number:

$$2 \frac{1}{4} =$$

$$1 \frac{1}{4} =$$

$$2 \frac{3}{4} =$$



Write the improper fractions as mixed numbers.

*Example: Improper fraction  $\frac{27}{5}$*

*Divide the denominator into the numerator and rewrite the fraction as a sum of the whole part plus the remainder.  $\frac{27}{5} = 5 + \frac{2}{5} = 5 \frac{2}{5}$*

4. Write the improper fractions as mixed numbers

a)  $\frac{1}{8}$  of 26

b)  $\frac{1}{5}$  of 26

c)  $\frac{1}{9}$  of 26

d)  $\frac{1}{7}$  of 26

5. Add a fraction to make an equality correct:

a)  $\frac{1}{3} + \dots = 2$

b)  $\frac{1}{4} + \dots = 2$

c)  $\frac{1}{8} + \dots = 2$

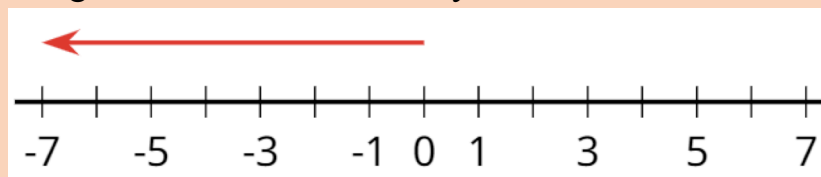
6. Jonathan picked  $2\frac{3}{4}$  pounds of peaches from the tree in his backyard. He gave  $1\frac{1}{4}$  pounds to his friend. How many pounds of peaches does Jonathan have left?
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7. How can you divide 17 apples between 4 children? How many apples (wholes and parts) each child will get?
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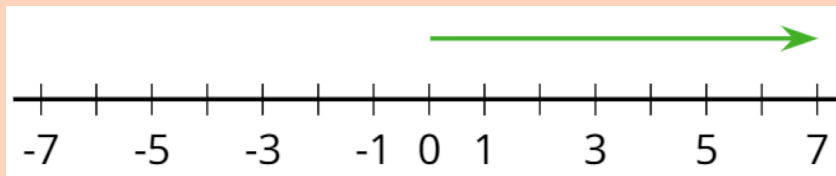
### NEW MATERIAL

#### Negative number

A negative number is a number that is less than zero. On a horizontal number line, negative numbers are usually shown to the left of 0.



**Positive number** - A positive number is a number that is greater than zero. On a horizontal number line, positive numbers are usually shown to the right of 0.



There are only two directions to go on the number line: **left and right**.

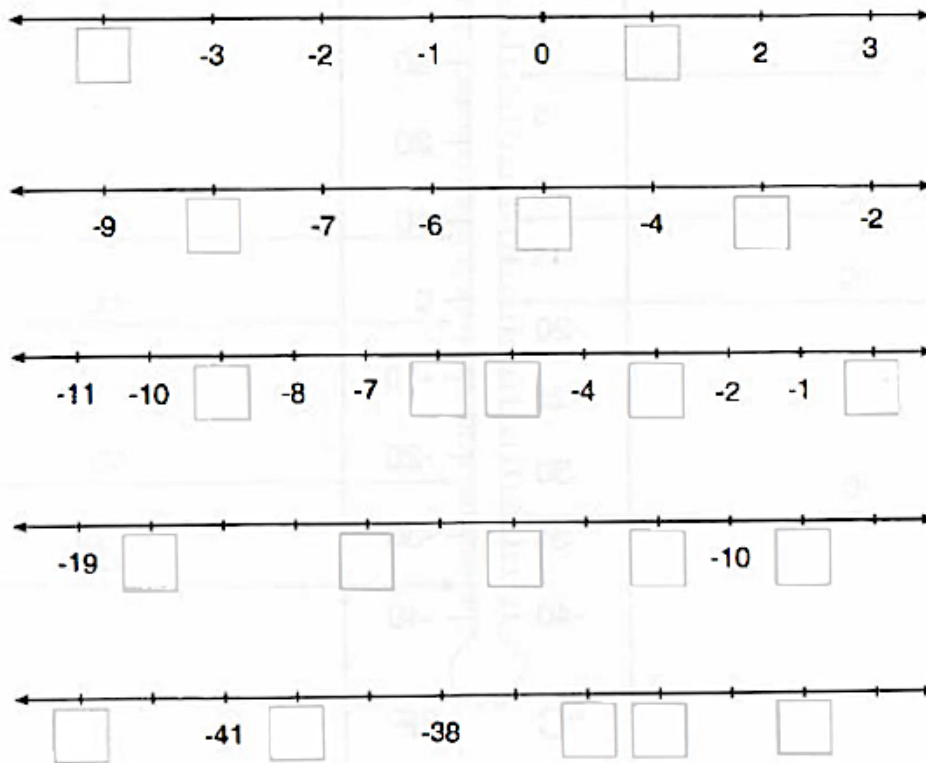
The negative sign preserves the distance from 0 but switches the direction from 0.

Negative numbers always have a “-” sign in front of the number. If a number has no sign it usually means that is a positive number.

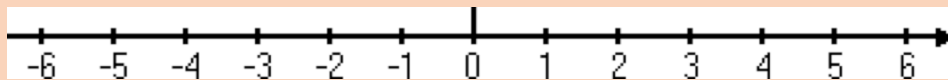
*Example:*  $-3$  is located three units to the left of zero. We know this because the number 3 tells us how many units from zero the number lies, and the sign of the number tells us which side of zero the number lies. In this case, the sign is negative so we will plot the number to the left of zero.

7.

Enter the missed numbers:



### Moving along the number line.



Note the arrowhead on the far right end of the number line above. That arrow tells you the direction in which the numbers are getting bigger. In particular, that arrow also tells you that the negatives are getting *smaller* as they move off to the left. That is,  $-5$  is *smaller* than  $-4$ .

8.

Compare, using  $<$ ,  $>$  or  $=$ .

a)  $-3$  \_\_\_\_\_  $6$

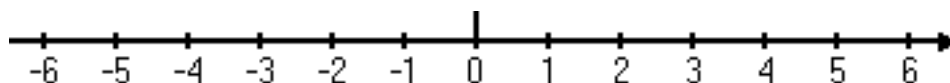
b)  $-3$  \_\_\_\_\_  $-6$

c)  $0$  \_\_\_\_\_  $1$

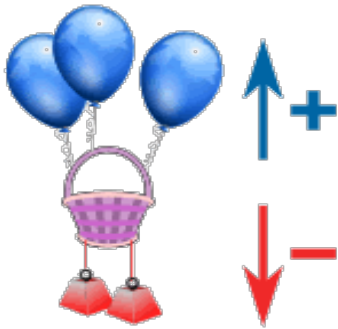
d)  $0$  \_\_\_\_\_  $-1$

e)  $-2$  \_\_\_\_\_  $2$

f)  $-2$  \_\_\_\_\_  $-3$



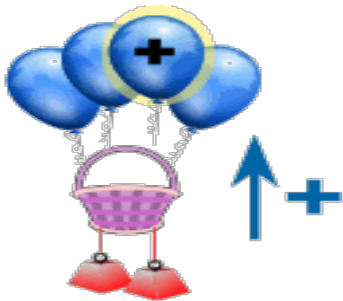
### Adding a negative number.



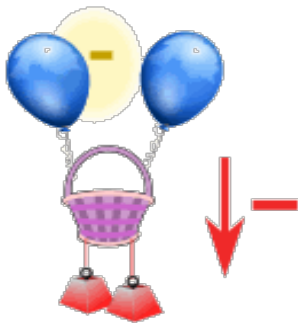
Let's think of the balloons as of positive numbers. Adding balloons will pull basket upwards.

Let's think of the weights as of negative numbers. Adding up more weights will pull basket downwards.

- The balloons pull up (positive direction)
- The weights drag down (negative direction)



Adding a positive number is a simple addition and means that we add balloons. The basket gets pulled upwards (positive direction)



Subtracting a positive number is just simple subtraction - we can take away balloons and the basket gets pulled downwards (negative direction)

$$\text{Example: } 6 - 3 = 3$$



Adding a NEGATIVE number can be shown as adding weights.

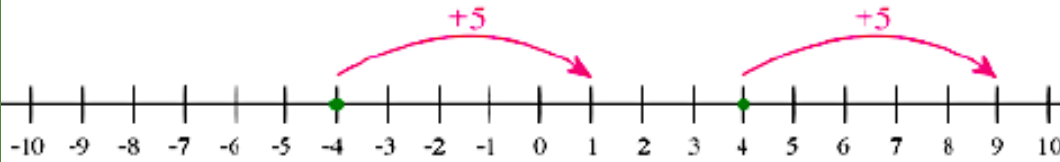
$$\text{Example: } 6 + (-3) = 3$$

The last two examples showed us that taking away balloons (subtracting a positive) or adding weights (adding a negative) both make the basket **go down**.

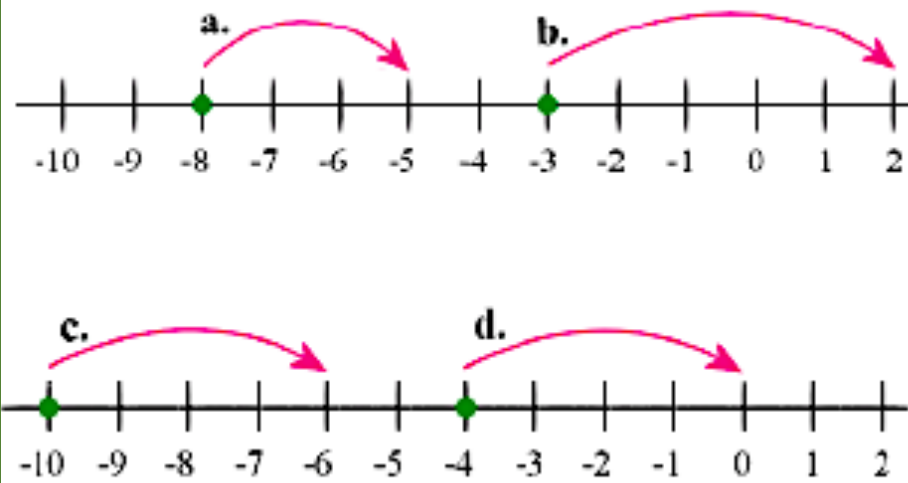
**Subtracting a positive number is the same as adding a negative number.**

**9.** To add number 3 to any number, we start at the number and move 3 units on to the right. To add  $(-8) + 3$ , we start at  $(-8)$  and move 3 units to the right.

a) Write an addition sentence to match the number line jump:

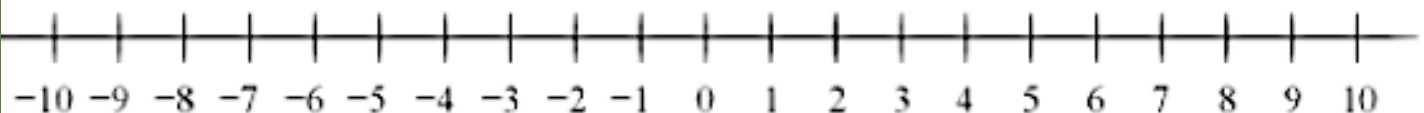
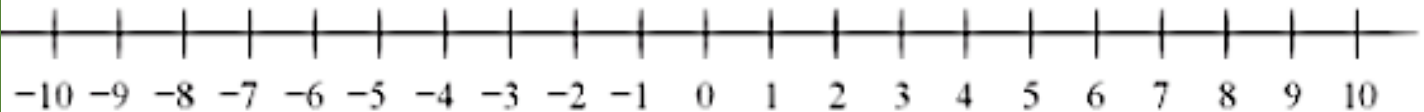


b) Write an addition sentence to match the number line jump:



**10** Draw the number line jump for each addition sentence and find a value:

- |               |               |               |                 |
|---------------|---------------|---------------|-----------------|
| a) $-8 + 2 =$ | b) $-4 + 9 =$ | c) $-7 + 5 =$ | d) $-10 + 12 =$ |
| e) $2 - 8 =$  | f) $9 - 4 =$  | g) $5 - 7 =$  | h) $10 - 12 =$  |



## Did you know ...

**Negative numbers**

Adapted from the article by Leo Rogers

Negative numbers have a long and sometimes contentious history. Mathematicians on the Indian subcontinent had been using negative numbers for a thousand years before Europeans got around to accepting the idea. (We owe our number system, including the number zero, to Indian mathematicians, too.) Chinese mathematicians beat the Europeans by two millennia in using negative numbers. (Humorous note: When the Indians were writing, they used the "plus" sign, "+", to indicate negative numbers.)

In **India**, negative numbers did not appear until about 620 CE in the work of Brahmagupta (598 - 670) who used the ideas of 'fortunes' and 'debts' for positive and negative. By this time a system based on place-value was established in India, with zero being used in the Indian number system. Brahmagupta used a special sign for negatives and stated the rules for dealing with positive and negative quantities as follows:

*A debt minus zero is a debt.*

*A fortune minus zero is a fortune.*

*Zero minus zero is a zero.*

*A debt subtracted from zero is a fortune.*

*A fortune subtracted from zero is a debt.*

*The product of zero multiplied by a debt or fortune is zero.*

*The product of zero multiplied by zero is zero.*

*The product or quotient of two fortunes is one fortune.*

*The product or quotient of two debts is one fortune.*

*The product or quotient of a debt and a fortune is a debt.*



Europeans were not alone in being ignorant of, or dismissive of, negative numbers. Egyptians, nearly two thousand years ago, regarded negative numbers as being ridiculous. Europeans, like the Egyptians, used a mathematics based on geometrical concepts such as area, which is always positive. This retarded their mathematical progress, because they were thinking of numbers in an unhelpful way. However, when European scholars started translating Arabic texts obtained from North Africa, they were finally exposed to new ways of thinking, and started catching up.

As we know, practical applications of mathematics often motivate new ideas and the negative number concept was kept alive as a useful device by the Franciscan friar Luca Pacioli (1445 - 1517) in his *Summa* published in 1494, where he is credited with inventing double entry book-keeping.

It was not until the 19th century when British mathematicians like De Morgan, Peacock, and others, began to investigate the 'laws of arithmetic' in terms of logical definitions that the problem of negative numbers was finally sorted out.

### Uses of negative numbers:

- A building elevator – Consider a building with an elevator which goes up and down. When it is above ground it is a positive height. When an elevator goes down into the basement, it is now below ground and the height becomes negative.
- Sport
- Science.
- Finances - in bookkeeping, amounts owed are often represented by red numbers, or a number in parentheses, as an alternative notation to represent negative numbers.
- Find more examples of using negative numbers in our lives.

