Math 3, Classwork 9

WARM-UP

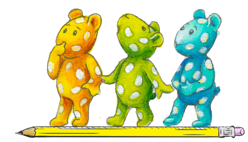
1. Compare using $<$, $>$, $=$

$a + b$ ____ $b + a$ $a + a + a + a$ ____ $3 \times a + a$ $a + 17$ ___ $17 + a$

$4b - b - b$ ____ $3 \times b$ $a + 1 - n$ ____ $a + 1 + n$ 4×4 ____ 8×2

7×3 _____ 6×4 $7 \times a$ _____ $7a$ $71 - 25$ ___ $72 - 26$

2. Write down the expressions:



a) Julia computes the perimeter of a rectangle by adding the length, *l*, and width, *w*, and doubling this sum. Victoria computes the perimeter of a rectangle by doubling the length, *l*, doubling the width, *w*, and adding the doubled amounts. Write an expression for Julia's way of calculating the perimeter. Write an expression for Victoria's way as well.

Julia's: ____

Victoria's ____

b) Milan spent $*a* for a soccer ball. It was $14 less than he spent for his soccer cleats.

Write and expression for a cleats's price.

____

c) There are *b* boys in the class who play soccer, *c* boys in the class who play tennis and 4 boys who don't do any sport. Write an expression for a total number of boys in the class.

____

d) The distance between your house and a school's bus stop is *a* meters, the distance between bus stop at school and your class is *b* meters. What is the distance you walk every day on your way to and from school?

____

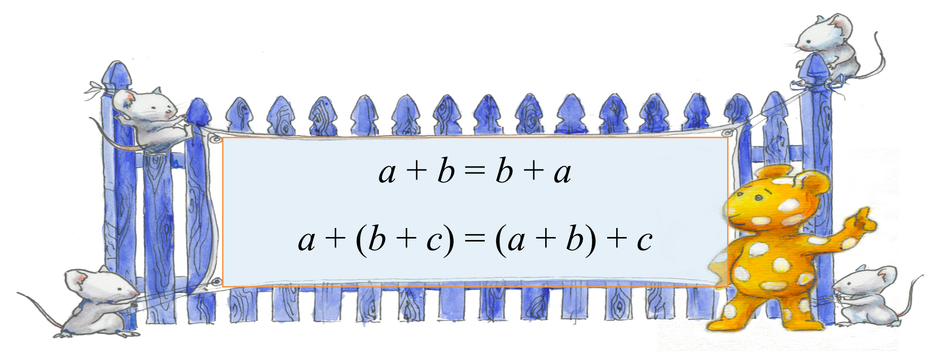
REVIEW I

Commutative and Associative properties of addition

Commutative property: When two numbers are added, the sum is the same regardless of the order of the addends. For example: 3 + 5 = 5 + 3

The property works for any number of addends.

Associative property: When three or more numbers are added, the sum is the same regardless of grouping of the addends. For example: (3 + 5) + 1 = 3 + (5 + 1)



3. a) Calculate using commutative property of addition (a + b = b + a):

7 + 16 + 3 = 11 + 8 + 9 = 7 + 6 + 7 =

48 + 37 + 12 + 13 = 50 + 29 +21 = 42 + 8+58 + 92 =

b) Calculate using associative property of addition:

(13 + 72) + 7= (85 + 18) = 15 =

328 + (22 + 650) = 117 + (725 + 23) =

REVIEW II

Area and units of area

Area is a measure of how much surface is covered by a particular object or figure. The square with a unit side is used as a unit of measure for area.

Every unit of **length** has a corresponding unit of area, namely the area of a square with the given **side length**. Thus areas can be measured in square meters (m^2), square centimeters (**cm^2**), **square millimeters** (mm^2), square kilometers (km^2), square feet (ft^2), square yards (yd^2), square miles (mi^2), and so forth.

All the dimensions must be in the same units.

Two sheets of paper have twice the area of a single sheet, because there is twice as much space to write on.

Different shapes have different ways to find the area. For example, in a rectangle we find the area by multiplying the length times the width. In the rectangle on the right, the area is 2×3 or 6. If you count the small squares you will find there are 6 of them.

a) $2 \times 3 = 6$ b) $3 \times 2 = 6$

NEW MATERIAL

Properties of Multiplication

The commutative property for multiplication states that when you multiply two or more numbers together, the order in which you multiply them will not change the answer. Using symbols, you can express this rule by saying that, for any two numbers $\boldsymbol{a}$ and $\boldsymbol{b}$, $\boldsymbol{a \times b = b \times a}$. This could also be expressed for three numbers, ***a, b and c***, as $\boldsymbol{a \times b \times c = a \times c \times b = c \times b \times a}$ and so on. As an example, 2×3 and 3×2 are both equal to 6.

The **associative property** says that the grouping of the numbers does not matter when multiplying a series of values together. Grouping is indicated by the use of brackets in math and the rules of math state that operations within brackets are to take place first in an equation. You can summarize this rule for three numbers as $\boldsymbol{a \times (b \times c) = (a \times b) \times c}$. An example using numerical values is 3 x (4 x 5) = (3 x 4) x 5, since 3 x 20 is 60 and so is 12 x 5.

The **distributive property** holds that a term consisting of the sum (or difference) of values multiplied by a number is equal to the sum or difference of the individual numbers in that term, each multiplied by that same number. The summary of this rule using symbols is that

$\boldsymbol{a \times (b + c) = a \times b + a \times c}$, or $\boldsymbol{a \times (b - c) = a \times b - a \times c}$.

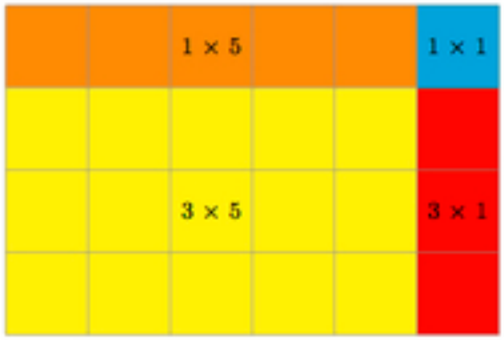
An example could be 2 x (4 + 5) = 2 x 4 + 2 x 5, since 2 x 9 is 18 and so is 8 + 10.

4. Find $(3+1) \times (5+1)$ using the distributive property.

a) Using the distributive property several times we find

$(3+1)\times(5+1) = 3\times(5+1)+1\times(5+1) = (3\times5+3\times1)+(1\times5+1\times1) = (15+3)+(5+1) = 24.$

Below the four rectangles make up the larger 4 by 6 rectangle. Each of the four rectangles is labeled with its own color.



The four labeled rectangles do not overlap, so the sum of their areas is the area of the large rectangle. This tells us that $4\times6=3\times5+3\times1+1\times5+1\times1$ as we also saw in part (a).

5. Use distributive property to do multiplication

(*Example:* $13\times6 = (10+3)\times6 = 10\times6 + 3\times6 = 60+18=78$ *or*

$9\times25 = (10-1)\times25 = 10\times25 - 1\times25 = 250 – 25 - 225$

$14\times8 =$ ____

$16\times5 =$ ____

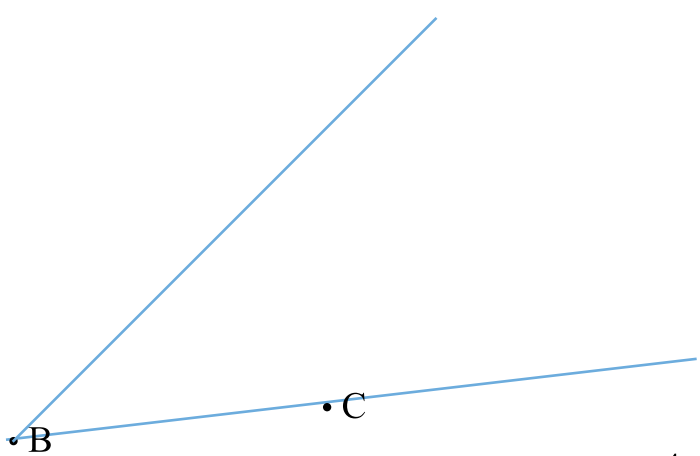
$102\times7 =$ ____

$12\times25 =$ ____

$110\times4 =$ ____

$19\times5 =$ ____

6. Use a compass to find a point A on the other side of the angle so that point A will be on the same distance from the vertex of the angle – B, as point C is.



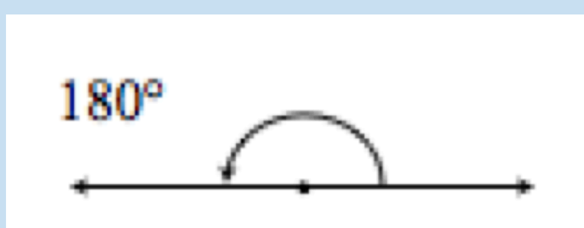
7. Using a compass and a straightedge (ruler) construct a line segment which is

a) The sum of the segments *a* and *b*.

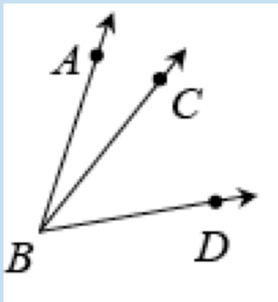
b) The difference of those segments.

REVIEW III

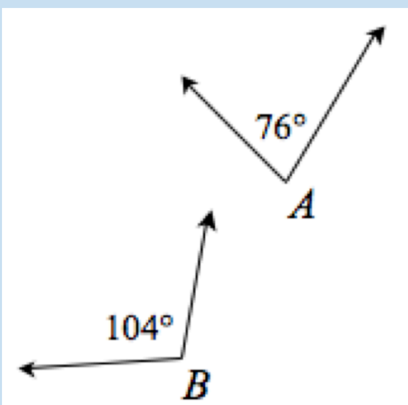
A **straight angle** is an angle that forms a straight line. It measures 180 degrees.

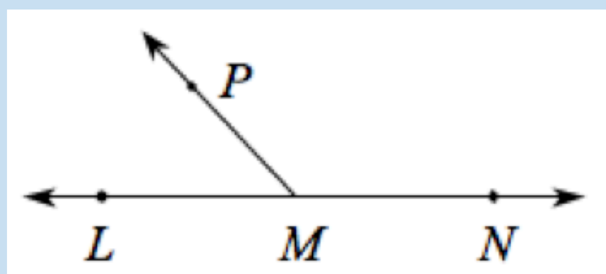


Adjacent angles: Two **angles** are **Adjacent** when they have a common side and a common vertex (corner point) and don't overlap. In the example at right, ∠ABC and ∠CBD are adjacent angles.



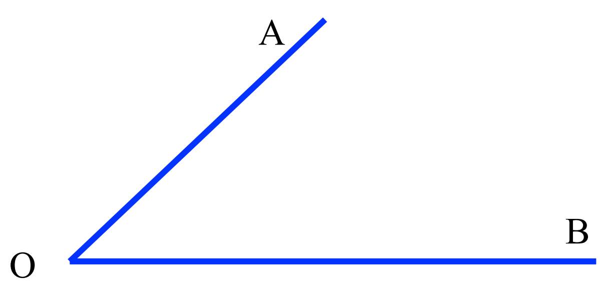
Supplementary angles: Two angles A and B for which A + B = 180°. Each angle is called the supplement of the other. In the example at left, angles A and B are **supplementary**. Supplementary angles are often adjacent. For example, since ∠LMN is a straight angle, then ∠LMP and ∠PMN are supplementary angles because ∠LMP + ∠PMN = 180°.





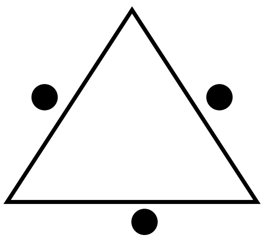
8. From the vertex of angle AOB draw a ray OK so that it forms obtuse angles with both sides – OA and OB. Use a protractor to measure those angles and write down the results.

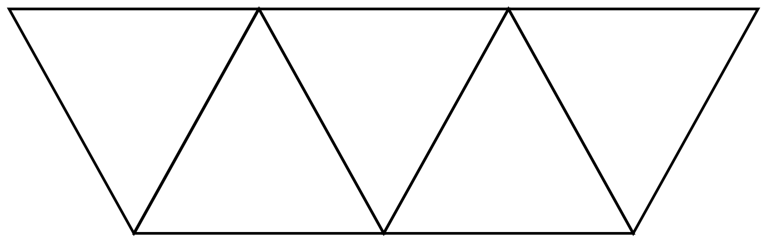
∠KOA = ___________________ ∠KOB = _______________________



Challenge yourself

10. A classroom has triangular tables. There is enough space at each side of a table to seat one child. The tables in the class are arranged in a row (as shown in the picture below).





- How many children can sit around 1 table? ___________
- Around a row of two tables? _______________________
- Around a row of three tables? _____________________
- Find an algebraic expression that describes the number of children that can sit around a row of *n* tables. Explain in words how you found your expression.

____

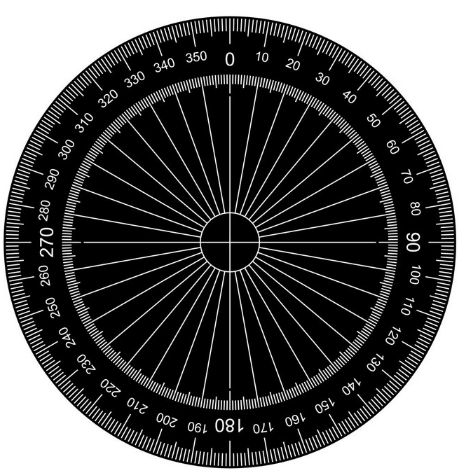
If you could make a row of 25 tables, how many children would be able to sit around it?

____

Did you know …

A full circle is 360 degrees, but why?

You must be wondering what mathematical reasons there might be for using 360 degrees to represent a complete circle.



1. Mathematical reasons (Theory # 1):

The number 360 is divisible by every number from 1 to 10, aside from 7. It actually divides into 24 different numbers: 1,2,3,4,5,6,8,9,10,12,15,18,20,24,30,36,40,45,60,72,90,120,180 and 360 itself. These 24 numbers are called the divisors of the number 360. This is the highest number of divisors for any positive whole number up to its own value of 360.

This characteristic of the number 360 makes it a **highly composite number**. Numbers are said to be highly composite if they are positive integers with more divisors than any smaller positive integer has. The only highly composite numbers below 360 are 2, 6, 12, 60 and 120. Highly composite numbers are considered good base numbers with which to perform common calculations. For example, 360 can be divided into two, three and four parts and the resulting number is a whole number. The resulting numbers are 180, 120 and 90.

2. The length of a year (Theory #2):

Have you all ever wondered why there are exactly 365 days in a year? Again why wouldn't they use a more convenient number like 300 or 400? Ancient astronomers, mainly the Persians and the Cappadocians, noticed that the sun took 365 days to come back to the exact same position. For simplicity, they decided to round that down to 360 days per year.

In other words, the sun advances by one degree each day along its elliptical path. The Persians had a leap month every 6 years to adjust for the 5 extra days. Also, the lunar calendar has a total of 355 days, while the solar calendar has 365. And what number sits perfectly between the two and is a highly composite number?
Yes… 360!

3. Historical reasons (Theory #3):

Another theory that suggests why a full circle is considered to be 360 degrees comes from the Babylonians. The Sumerians and Babylonians were known to use the **Sexagesimal** numeral system. The sexagesimal system is one with a base value of 60, whereas the current system we use is known as the decimal system and has a base value of 10. So, once we reach the 10th number, we start repeating the symbols (of previous numbers, from 0 to 9) to form new numbers.
The Babylonians had 60 different symbols with which they formed numbers. Again, why would they use 60? Because 60, just like 360, is a highly composite number with up to 12 factors. Just as we can count 10 on our fingers for the decimal system, we can also count to 60. Start by counting the knuckles of the 4 fingers (not the thumb) on your right hand. 12, right? Now, on the other hand, raise any of those fingers to remember that you finished one iteration and got the number 12. Now, repeat the same procedure as many times as the number of fingers remaining on the left hand. The number you will end up with is 12 knuckles x 5 fingers = 60.