

## MATH BATTLE!

FEB 23, 2020

1. A teacher writes 10 natural numbers on the board. Show that it is always possible to choose some of them and put signs + and - between them so that the resulting expression is a multiple of 1001.  
[Hint: pigeonhole principle!]
2. A plot of land has the shape of a rectangle with sides 10 meters and 40 meters. It is known that this plot is crossed by a buried water pipe, going in a straight line. A plumber wants to find the pipe, by digging narrow trenches until one of them hits the pipe.  
What is the shortest total length of trenches he has to dig to guarantee that he will find the pipe? One option is digging the trenches along two diagonals of the square. Can you do better than that?  
[For this problem, you can use calculators if necessary.]
3. Find the smallest positive integer  $n$  such that
  - $n$  is divisible by 80
  - It is possible to permute two (different) digits of  $n$  so that the resulting number is also divisible by 80
4. Three runners are doing laps on a round track of length 300 m. Each runner runs at constant speed; they run in the same direction, but all speeds are different.  
A photographer standing in the center of the track wants to take a photo which would show all of the runners.
  - (a) Show that if the photographer has a camera which can cover  $90^\circ$  angle (i.e., cover 75 meters of track) or more, then he can always get all 3 runners in a single shot, regardless of their speeds and initial positions.
  - (b) Show that if his camera can only cover less than  $90^\circ$  angle, then, for some choice of runners' speed and initial positions, he would never be able to get them in a single shot.