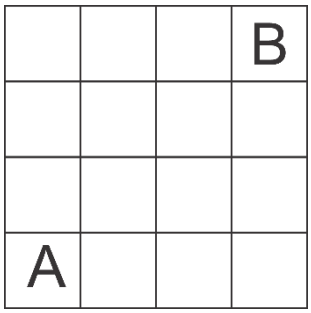
Algebra and Geometry 2. Homework 8.

Please, send me your homework by mail (scan or photo).

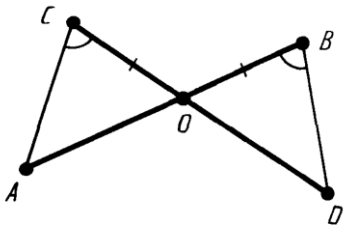
The entry in the *n*th row and *k*th column of Pascal's triangle is denoted

$$C(n,m) = \binom{n}{m} = \frac{n!}{m!\,(n-m)!}$$

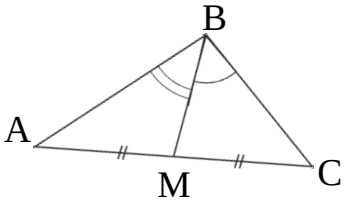
1. 10 points are marked on a plane, no 3 of which lie on the same line. How many triangles are formed by these points? By 50 points?
2. How many ways are there to go from box A to box B in the table moving always only to the right or up?
   Same problem for 10x10 table?



3. You are creating a 4-digit pin code. How many choices are there in the following cases?
   a. With no restriction.
   b. No digit is repeated.
   c. No digit is repeated, digit number 3 is a 0.
   d. No digit is repeated, and they must appear in increasing order.
   e. No digit is repeated, 2 and 5 must be present.
4. On two parallel lines a and b, points $A_1, A_2, ..., A_m$ and $B_1, B_2, ..., B_n$ are selected, respectively, and all segments of the form A_iB_j are drawn ($1 \le i \le m$, $1 \le j \le n$). How many intersection points will there be if it is known that no three of these segments intersect at one point?
5. Segments [AB] and [CD] intersect at point O. Prove that triangles ACO and DBO are congruent if angle $\angle ACO$ is congruent to angle $\angle OBD$ and segments [CO] and [OB] are congruent.



6. *(difficult problem) Prove the congruency of 2 tringles by median and angles between median and sides. (Two triangles are congruent if medians and angles formed by these medians and sides are congruent). (Hint: continue the medians to its length).



7. Factorize:
   Example:

$$\sqrt{x} + x = \sqrt{x} + \sqrt{x^2} = \sqrt{x}\left(1 + \sqrt{x}\right)$$

   $a.\ a - \sqrt{a};$ $\quad b.\ \sqrt{a^3} + 2a;$ $\quad c.\ 3mn - \sqrt{m^3n^2}$
8. Prove the identity

$$\left(\frac{a}{b^2+ab} - \frac{a-b}{a^2+ab}\right) : \left(\frac{b^2}{a^3-ab^2} + \frac{1}{a+b}\right) = \frac{a}{b} - 1.$$

9. Think about math topic you want to study in our class. List them.