How to find all the currents and voltages in any circuit.

Last class we have discussed a general way how we can calculate all the currents and voltage drop in any circuit of voltage sources and the resistors. The circuit in Figure 1 has two voltage sources, so calculating the currents may seem confusing and difficult. But, in fact, it is not. We can do that using two simple rules and the Ohm's law. These rules are:

1. Sum of the currents in any circuit node is zero. Currents flowing into the nodes and out of the nodes should have different sign.

The explanation of this rule is quite simple: the charge which flows into the node has to be equal the charge which flows out. Otherwise, the charge of the node would increase infinitely with time.

2. Sum of the voltage drops along any closed loop of the circuit has to be zero. You can choose the direction (clockwise or counterclockwise – again it is arbitrary). Then, moving along the loop in the direction you have chosen, you sum up all the "voltage drops" along the loop. The sign of the voltage drop or the voltage source must be chosen according to the following way: the voltage on a resistor is positive if you are moving along the current and negative if you are moving against the current. The voltage of the voltage source is positive if , going around the loop you meet "plus" first.

*If we assume that the sum of the voltages around a close loop is nonzero and equals ΔU, then as the charge **q** moves around this loop and returns to the starting point it earns the potential energy P=q ΔU. But the potential energy depends only on the position of the object and does not depend on the way we use to bring the object to this position, so ΔU has to be zero.*

These rules are known as ***Kirchhoff circuit laws***. They named after German physicist Gustav Kirchhoff.

Gustav Kirchhoff (1824-1887)

As you know the currents, you can calculate all the necessary voltages using Ohm's law. But before I will introduce two new terms:

Nontrivial node - is the connection of 3 or more wires.

Branch – part of the circuit which connects two nontrivial nodes. Branch is the group of elements connected in series. All circuit elements in the branch share same current.

There are several steps you can use to apply the Kirchhoff laws for the circuit solution. These steps may seem a bit scary, but in reality they are simple as long as you will read them attentively:

1. Circle each nontrivial node of the circuit. Imagine that you know potentials of all the nontrivial nodes. Then, using Ohm's law you will be able to find currents in all the branches and the circuit will be solved. Moreover, we do not even need to know the potentials. Instead, we need the *potential difference* between any two nodes. So, adding or subtracting the same number from all the potentials will not change the currents since the potential differences (voltages) will stay the same. For example, we can subtract the number which is equal to one of the node potentials. Then, potential in this node will be zero. So, we can arbitrary choose one node and make its potential equal to zero. This will be our **reference node** (or **ground node**).
2. Arbitrary choose the reference node. The potentials of the other nodes will be our variables – we have to find these values.
3. Mark the direction of the current in each branch of the circuit. Definitely, we do not know how do the currents flow in reality – we have not solved the circuit yet. But at this point we may choose the direction of the currents **arbitrary**. As long is you will obtain a negative value for the current in the end, it just means that in reality the current flows opposite to the direction you have chosen.
4. Write Kirchhoff current law for each of the nontrivial nodes except the reference node. This law state that sum of the currents which meet at the node is equal to zero. Currents flowing into the node and currents flowing out of the node should have different signs (for example let us take the currents flowing into the node as positive). As the result you will have the equation for each nontrivial node except the reference one.
5. Express all the branch currents through the node potentials using Ohm's law and plug into your Kirchhoff current law equations.
6. Solve the equations and find the node potentials.
7. Plug the obtained node potentials in the expressions for the currents (paragraph 4) and calculate the currents. The circuit is solved. Now you know all the currents and all the voltages in the circuit.
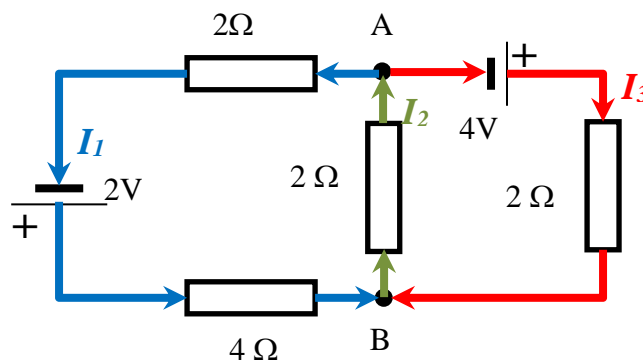
Below is an example:

Figure 1.

1. In the circuit shown in figure 1 there are 2 nodes: A and B and 3 branches.

2. The directions of currents are I_1 (blue), I_2 (green) and I_3 (red) are chosen arbitrary.
3. We will choose node B as reference, the potential of this node V_B is equal to 0. So, we have to find just one number: potentioal of node A: V_A.
4. Kirchhoff current law for node A:
$$I_2 - I_1 - I_3 = 0 \qquad (1)$$
   I_2 and I_3 are taken negative since they go out of the node.
5. Then express all the currents through the node potentials. Here I will remind that the voltage between two nodes, say A and B, is the difference of potentials in these nodes. Let us express the currents through the potentials of the nodes. Let us start from current I_2. The current flows from node B to node A through 2 Ohm resistor. So, the current can be expressed as:
$$I_2 = \frac{V_B - V_A}{2\,Ohm} = \frac{0 - V_A}{2} = -\frac{V_A}{2} \qquad (2)$$
Then, current I_3 is equal:

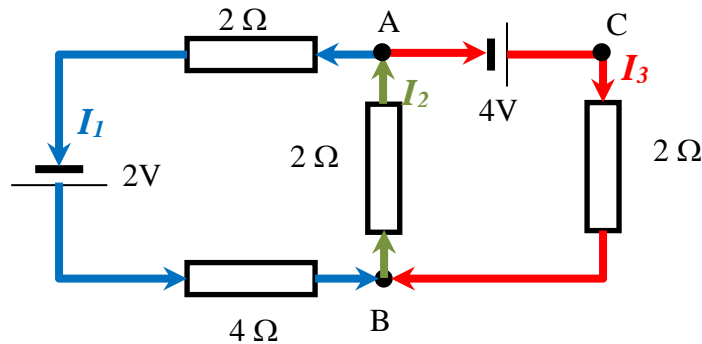$$I_3 = \frac{V_C - V_B}{2\,Ohm} \qquad (3)$$

Figure 2.

But potential in point C is 4 Volt higher than this in point A since there is 4V voltage source is connected between these points. Then, $V_C = V_A + 4$ and

$$I_3 = \frac{V_A + 4}{2} \qquad (4)$$

Finally, let us express I_3. We can use Kirchhoff voltage low: sum of the voltages along the loop formed by the blue and green brunches is equal to 0:
$$(V_B - V_A) + I_1 \cdot 2\,Ohm - 2V + I_1 \cdot 4\,Ohm = 0 \quad (5)$$
Here we used the sign rule:
Sign of the voltage drop across a resistor: the side where the current enters is plus, the other side is minus. So, if you are "moving" along the current, the voltage is positive, otherwise it is negative.
Sign of the voltage source: if , going along the loop, you "meet" "plus" side of the voltage source this voltage has to be counted as positive, otherwise it is negative. That is why we

have included 2V source as a negative one. We have obtained equation (5) by moving counter-clockwise along the loop (along the current directions).

From (5) we will express I_1:

$$I_1 = \frac{V_A + 2}{6} \qquad (6)$$

Then we can plug expressions (2), (4) and (6) into (1):

$$-\frac{V_A}{2} - \frac{V_A + 2}{6} - \frac{V_A + 4}{2} = 0 \qquad (7)$$

6. Solving equation (7) we can find: $V_A = -2V$
7. Plugging this numbers in the expressions for the currents we have:

$$I_1 = \frac{V_A + 2V}{6\ Ohm} = \frac{-2V + 2V}{6\ Ohm} = 0$$

$$I_2 = -\frac{-2V}{2\ Ohm} = 1A$$

$$I_3 = \frac{-2V + 4V}{2\ Ohm} = 1A$$

Interesting that in this circuit the current will not flow in the blue branch at all! Both I_2 and I_3 are positive. It means that we have occasionally guessed the right directions for these current.

Problem:

Find voltage between the points A and B.