Homework 18

Last class we discussed the phenomenon of resonance. Let us take a simple oscillator – our favorite mass, attached to a coil spring on frictionless table. Now, we will apply a small periodic force to the mass. The force is directed along the axis of the coil spring. Simply speaking, we will slightly push the mass back and forth, so our force is changing in time as follows:

$$F = f\,\sin(\Omega t) \quad (1),$$

here $\boldsymbol{f}$ is the amplitude of the force and $\boldsymbol{\Omega}$ is angular frequency of the force. Such system is called "driven oscillator". If the angular frequency $\boldsymbol{\Omega}$ of our driving force (i.e. the number of pushes per 2π seconds) is much less than the angular frequency of our oscillator $\omega_0 = \sqrt{\frac{k}{m}}$, the mass is just following the force, oscillating at a frequency $\boldsymbol{\Omega}$. The amplitude of oscillations in this case is: $A = \frac{f}{k}$. Just to remind, here $\boldsymbol{f}$ is the amplitude of the driving force, k is the elastic constant of the spring.

If the angular frequency of the driving force is much higher than the oscillator frequency, $\Omega \gg \omega_0$, the driving force is changing in time so fast that the mass has no time to be sufficiently accelerated between the "opposite" pushes. In this case our oscillator stays still and the amplitude of oscillations is zero.

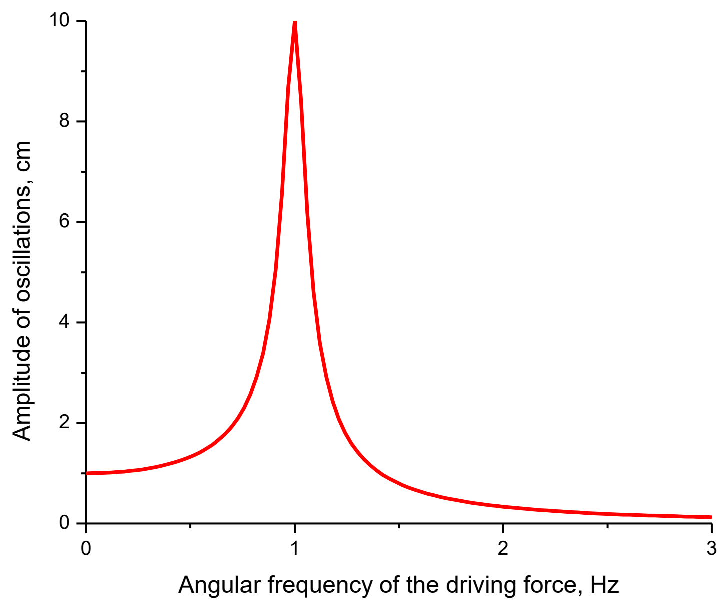


Figure 1. A typical dependence of the amplitude of driven oscillations on the angular frequency of the driving force.

If we drive the oscillator with the frequency which is exactly equal to the frequency of the oscillator$\Omega = \omega_0$, we will see that the amplitude of the oscillation is increasing in time and becomes large even if the amplitude of the driving force $\boldsymbol{f}$ is very small. Then, either the system breaks or the oscillation amplitude stabilizes at high level and stays high. The effect of dramatic increase of the oscillation amplitude as the frequency of the driving force is equal to this of the oscillator is called ***resonance***. A schematic dependence of the oscillation amplitude on the angular frequency of the driving force is presented in Figure 1 below. The oscillator frequency $\omega_0 = 1Hz$.

The plot in Figure 1 describes the behavior of a real system with friction. Friction is among the parameters which limit the maximum amplitude and determine the width of the resonance peak. Schematic time dependence of the mass displacement under resonant condition is shown in figure 2:

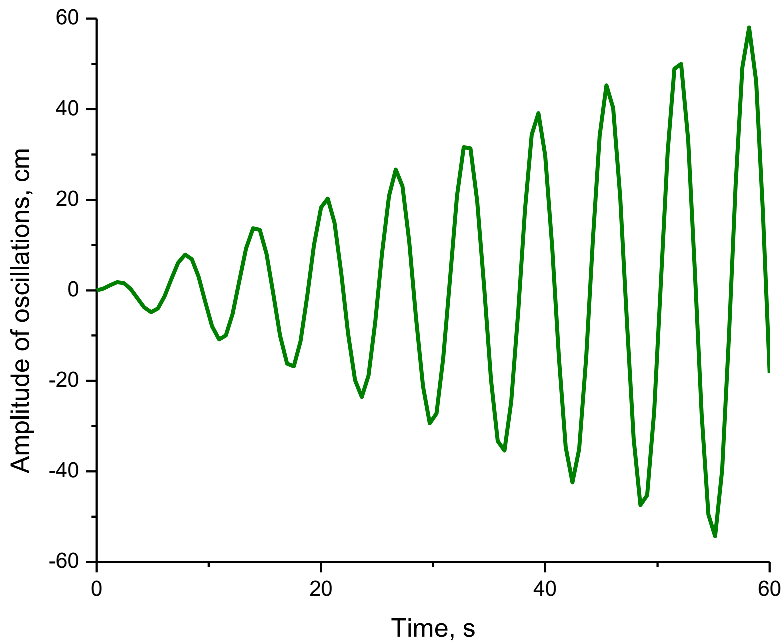


Figure 2. Schematic time dependence of the displacement of the oscillator mass at the conditions of resonant driving.

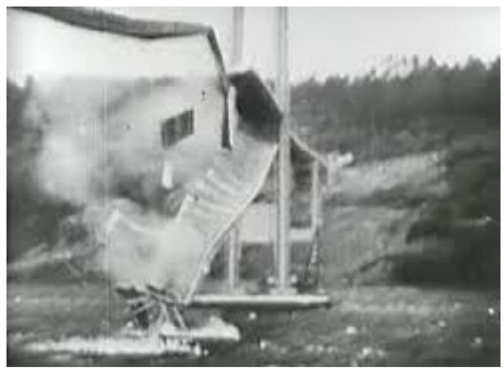
At resonant conditions the driving force always performs positive work on the oscillating mass increasing the mass's kinetic energy. This means that the driving force is in phase with the velocity of the mass, so the driving force is always directed along the mass's displacement. Since the frequencies of both the oscillator and the driving force do not depend neither on time nor the oscillation amplitude, the situation is stable and the oscillator's amplitude and kinetic energy increase steadily.

Outside the resonance - when the frequencies of the oscillator and the driving force are different - the latter spends some time in phase with the velocity of the load and some time – having opposite phase. As the driving force in phase with the load velocity, the load gets additional kinetic energy. As they have opposite phases, the energy is decreased because in this case the driving force decelerates the load. So the average kinetic energy is stable.

The phenomenon of resonance has extremely large application area in science and industry. In spectroscopy, various types of resonance allow us to determine chemical composition and structure of the objects from distant stars to human tissue (MRI). As any system in stable equilibrium, atoms and molecules have their own unique sets of resonant frequencies. Measuring these frequencies allows us to detect the presence of certain atoms and molecules.

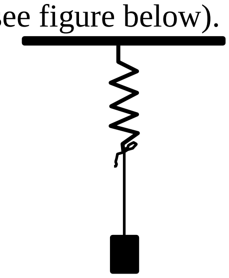
Another application of resonance is radio. Every time you turn the knob of your car's radio (or press the button, whatever…) to find your favorite radio station you use the resonance.

Sometimes resonant can be dangerous. The picture below shows the collapse of the Tacoma Narrows Bridge in 1940. The resonance vibration produced by the wind was identified as the reason of the collapse.



Problems:

1. I was driving on Park Avenue where the average distance between the dents on the road surface was 2m. The shock absorbers of my 1400kg car did not work properly. I noticed that if I go at a speed of 20 miles per hour my car starts shaking in the vertical direction so hard that I can hardly control the car's motion. Estimate the elastic constant of the coil springs (there are 4 of them) of the car's suspension.
2. A load attached to the end of the coil spring with the thread. Another end of the coil spring is attached to the ceiling (see figure below).



   Is it possible to arrange ***harmonic*** oscillation of the load with amplitude of 5cm and circular frequency of 6Hz?