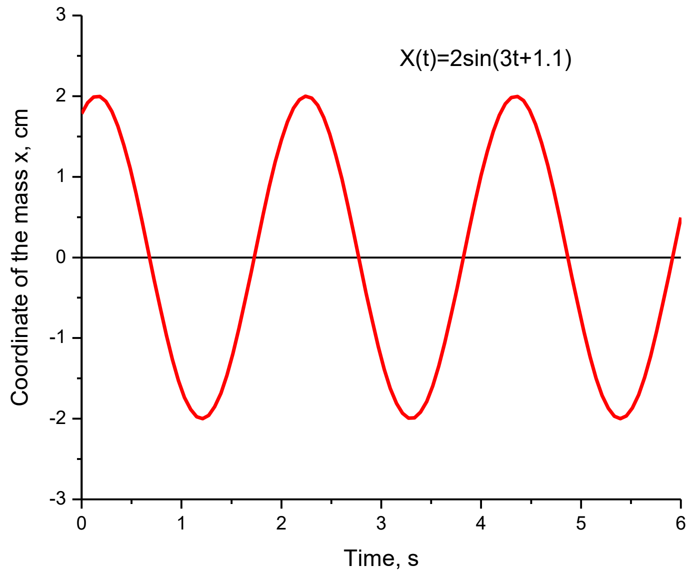
Homework 17

Let us consider a simplest oscillator: a 1kg mass attached to a coil spring fixed at a frictionless table. Coordinate x describes the displacement of the mass from the equilibrium position. The latter is taken as the origin (x=0). As long as the displacement changes with the time, it is a function of time x=x(t). To start the oscillations we displaced the mass and slightly pushed it. The dependence x(t) is shown in Figure below:



Here amplitude A=2cm; angular frequency (the number of oscillations per 2π seconds is 3, initial phase is 1.1rad.

Let us try to calculate maximum kinetic energy of the mass. As we remember, both the kinetic and the potential energy are changing as the mass is oscillating, but total energy remains the same. As the displacement of the mass is at maximum, the potential energy of the mass is at maximum as well, but the kinetic energy is zero because the velocity is zero. As the mass is passing the equilibrium point, its kinetic energy is at maximum and potential energy is zero. As long as the total energy remains the same for both positions, we can conclude that maximum kinetic energy equals to maximum potential energy. Potential energy of the deformed coil spring is:

$$E_{P\ max} = \frac{k \cdot A^2}{2} \quad (1)$$

Here *k* is elastic constant of the coil spring; A is the amplitude, i.e. maximum displacement from the equilibrium.

$$\frac{k \cdot A^2}{2} = \frac{m \cdot {V_{max}}^2}{2} \quad (2)$$

Here m is the mass, V_{max} is the magnitude of the maximum velocity or maximum speed. From (2) we can obtain:

$$V_{max} = \sqrt{\frac{k}{m}} \cdot A = \omega \cdot A \qquad (3),$$

Maximum velocity can be expressed through amplitude and angular frequency of the oscillator. This expression is general and can be applied to any harmonic oscillator. The maximum kinetic energy is :

$$E_{k\,max} = \frac{m \cdot {V_{max}}^2}{2} = \frac{m \cdot \omega^2 A^2}{2} \qquad (4)$$

So for our oscillator we have:

$$E_{k\,max} = \frac{1kg \cdot (3Hz)^2 \cdot (0.02m)^2}{2} = 0.0018\,J$$

1. Dependence of the coordinate of an object is described by the following formulae:
   a) $x = 3+20t-5t^2$
   b) $x=20t$
   c) $x=10\sin(6t+0.1)$

   Make plots and try to explain what type of motion is described by each graph.
2. For the function (c) of the problem 1 find amplitude, period and initial phase of the oscillations.
3. At the plot of the function c (problem 1) show the moments of time where the velocity is equal to zero.
4. Imagine a system with a stable equilibrium position (say, a pendulum). We are slightly pushing the bob. Is it possible that the system will not oscillate but just return to the equilibrium position? If yes, how we can arrange that?