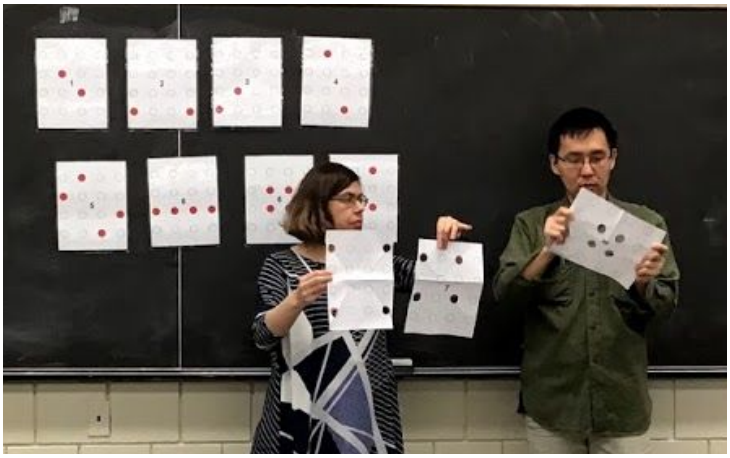
Math Club 2. Lesson # 19. **March 10, 2019**

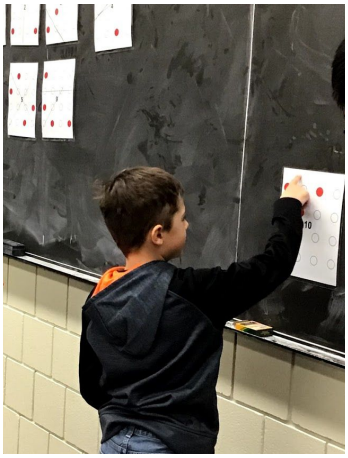
Punch My Dots

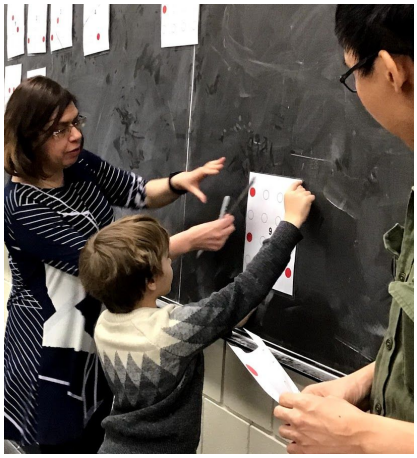
What is this and what does it do?

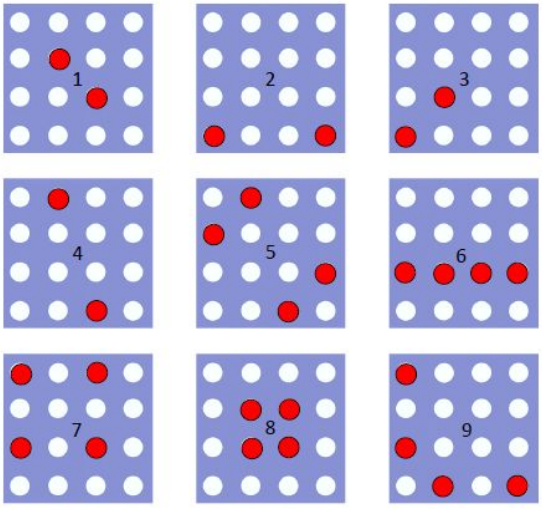
Yes, this is a hole-punch, and it makes holes on paper for us. If I fold my paper once, and punch it somewhere in the middle, what happens? Yes, I would get 2 holes. What if I fold my paper twice, make a hole, and unfold the paper? Would I get 3 holes or 4 holes? Why is it that I get 4 holes? Whenever I fold my paper, I double the number of layers of paper: 1 folding gives me 2 layers, another folding gives 2 layers for each layer that I already have. That's why we get the 4 holes: 2x2=4!

If I have 2 red dots arranged diagonally like this, can I make a single move and punch out both of them? Maybe by folding my paper so that both holes come together? Who can show me how to fold the paper? How about 2 holes like this, at the left and right sides at the bottom of the paper? Should I fold the paper along a line going through the middle from top to bottom?





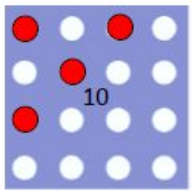


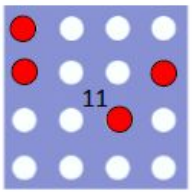


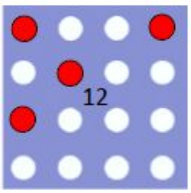
What if there are 4 dots? Can I make all dots come together with 1 fold? Of course not, I would need 2 folds! But how do we know where to fold the paper? We should always find the line that splits the dots (either 2 dots or 4 dots) into two equal parts that are mirrors of each other.

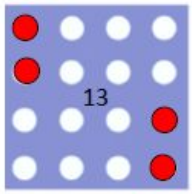
We should be folding the paper along its line of mirror symmetry!

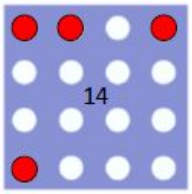
What do we do if the paper has dots on it that don't split into 2 halves that are mirrors of each other? Then we will try to reduce the number of holes by bring them together one by one. Try it yourself with the worksheet for this lesson.



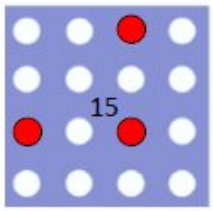


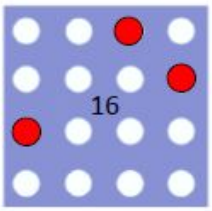


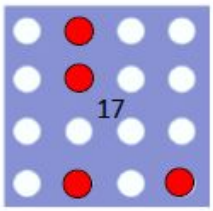




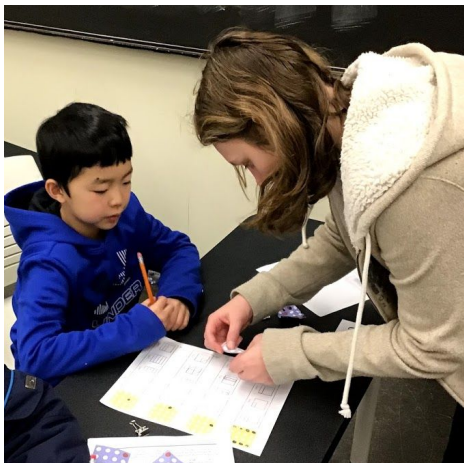
Now let us solve another, more difficult class of problems: for some arrangement, however we fold to reduce the number of holes, it seems like we are always folding onto the holes an area of paper that shouldn't have a hole. What do we do then? We might have to fold the paper so that certain parts, where we shouldn't make holes, will be hidden from the hole puncher. You will have to figure out yourself which way to "hide-fold" the paper to avoid punching holes where there shouldn't be one!

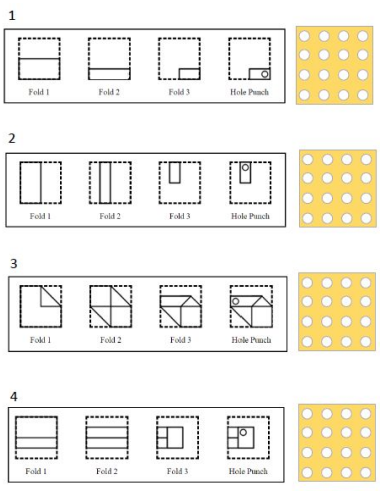




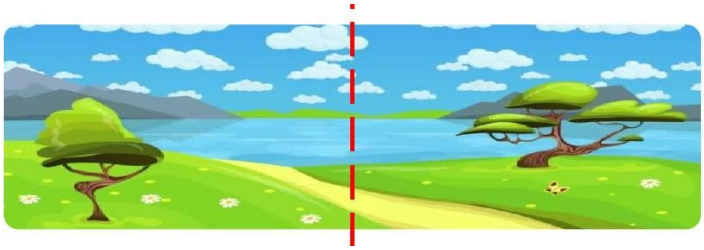


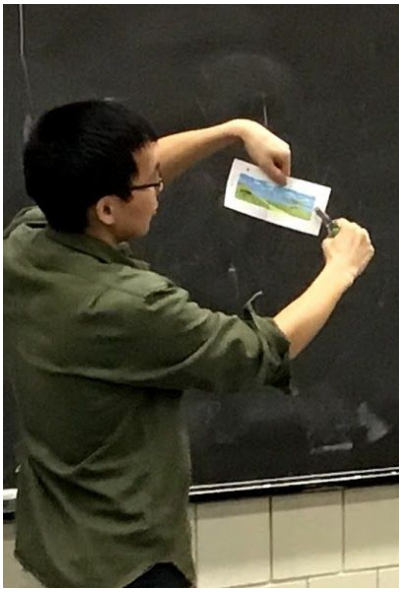
Now we will solve the opposite problem: what will be the hole arrangements if we fold the paper in a particular way and make a hole with one punch? Let us try problem number 1: we fold in half a rectangular sheet of 4-by-4 holes downward, then fold it downward again to get a 1/4th of a sheet. Then we fold it further in half to the right and get a small piece of paper. Then we will make a single hole on the right half of this small piece. Now, when we unfold this piece, how many holes will we see and where would they be? Let us try the problems ourselves: image the paper being folded and punched, guess where the holes would be, bubble them into the 4-by-4 grid next to it, and only then check if you are correct by actually folding and punching a real paper.





Let us play a game now! We will call it “Punch my dots!” We will have a paper field with a lake in the middle and a white boundary around it. The game will take place on this boundary with 2 colored pencils (one for each player) and hole punchers.





At turn one, player 1 will draw a dot using the pencil of his color, and player 2 will punch a hole in a place that is across the midline from the dot.

Once the hole is punched, the field will be folded along the midline and we check whether the hole is punched exactly where the dot is. If the dot is visible through the hole, then player 2 (the one who punched the hole) will score 1 point.

Then the players will switch: player 2 will draw the dot, and player 1 will punch the hole. When the paper is folded, and the dot is visible through the hole, then player 1 will score 1 point.