## Mirror Symmetry

Let us start with a very simple task: Lena will put on the board some arrangement of shapes and draw a line for a mirror. And the task will be to make the "mirror image" of Lena's arrangement on the other side of the mirror. If there is a hexagon touching the mirror, there has to be another hexagon on the other side of the mirror also touching it. If there are 2 squares attached to the hexagon, then there needs to also be 2 squares attached to the other hexagon, in the
 exactly opposite to the mirror sides.


Let us solve the reverse problem. I will put on the board an arrangement of shapes. And you will be drawing the line of symmetry of this arrangement. Remember that the line of symmetry has to divide the image exactly in half, so that for every shape on the left there is an identical shape, at an identical distance from the line on the right side of the line.

I have 4 circles on the whiteboard. They are arranged like this: red - in top left corner, yellow in bottom right corner, and 2 blue circles - in top right and bottom left corners. How can I then get an image of a single blue circle using a straight mirror?
We can simply put our mirror through the middle of one blue circle so that the mirror is facing away from all other circles. Then the mirror will reflect one half of this circle and along with the real half, there will be a full blue circle. And all other circles will be hidden because they are behind the mirror. They won't be visible to us and the mirror won't reflect them to us either. How about making 3 blue circles and 2 yellow circles? We will have to put the mirror so that the yellow circle, one full blue circle and one half of the other blue circle are on the mirror's one side so that they all get reflected and give us double of everything. This will make 2 full blue circles, 2 half blue circles (together making 1 more full blue circle),
 and 2 full yellow circles.

We have a large triangle with smaller triangles inside. There are green, blue, red and orange triangles, one of each, inside this large triangle. Now our task is to make the images of the 4 triangles given on the board (also large, with triangles inside, but with some colors repeating). How can we make a large triangle with a small red
 triangle at the top, and 2 small blue triangles at the bottom? What if we draw the "line of symmetry" for this image and look at the halves? It will go from top to bottom of the large triangle, and pass through the middle of the red and orange triangles, and separate the blue triangles. That means our mirror has to go through the middle of the red and orange triangles as well, and look towards the blue triangle.


I have a circle that is made of quarters of different colors. How can I get an image of a circle with 2 colors using a mirror? I can simply put my mirror along either of the two lines dividing the circle into halves. This would give us 2 half circles of different colors making up a full circle. How about making a circle of 3 colors? Is it possible? What if I put the mirror through the center of the circle, but not along any of the lines dividing it into quarters? I will have a mirror image with 2 quarters of the same color, and 2 more parts, each with a different color.

Can we make a circle with a single color using a single color then? Misha thinks it is impossible, because we need a half of a circle of a particular color to make a full circle of that same color. And all we have are quarters of a circle of a single color. And Lucas thinks that it
 is possible, but only with 2 straight mirrors! Each mirror will double our quarter and we will have 4 quarters altogether, making a full circle.

We now have a problem with the picture of a bird. We need to find an image that cannot be made from our bird picture using a single straight mirror. For each of the numbered images $1-8$, let us try to find where we need to put the mirror on our bird inside the blue frame to get it. Our task becomes somewhat easier if we try drawing the line of symmetry, the line dividing the image in half that are exact opposite of each other. Can we draw a line of symmetry through image 6?


Let's take a look at the pictures of the people with a ball: $\mathrm{A}, \mathrm{B}$ and C . We have to find which one of these people could be on the mirror images below. We will again find the like of symmetry of the mirror image, and see if we find the half on any of the pictures $\mathrm{A}, \mathrm{B}$, or C . It looks like the 1st image on the left was made from half of the person's head, an arm and a leg. But which of the arms look correct? Only people $B$ and $C$ have bent arms. So it can be them, and not person A.



Our next task is a very hands-on one: we have to draw and paint a real object based on its mirror images obtained from putting the mirror in various positions at various angles. The task isn't that easy, but let us try. It looks like the center of the image is a green circle with some colorful ornaments on it. How about the sides, left right, top and bottom? It looks like there is a yellow area with a red circle on it. There is a yellow loop attached to the object, and a sleeve-like thing as well. What could this be? Let us draw the lines of symmetry on all of them and see what the real object actually contains.

This problem is about coloring: we have a red fox. We know exactly what this real fox looks like, and we also have outlines of several of the fox's mirror images. Our task is to color the mirror images based on what the color of the real fox are. We can again start by drawing the lines of symmetry on the outlines, and see what the original fox's part looked like. Then we can find this part in fox's real image and copy the color.


