## Graphs: Show me your sister.

## Problem 1

There are three coins, one of them is fake (lighter than a real ones). How many weighings do we need to figure out which coin is fake?

Solution: Let's weigh any two first. If they are the same, the third coin must be a fake. If one of them is lighter, then that one is the fake. It took us only 1 weighing to find the fake one!

## Problem 2

There are 9 coins, one of them is fake (lighter than a real ones). How can we figure out for two weighings which coin is fake?

Solution: We split the coin into 3 groups of 3 coins each. We will then
 weigh 2 of the groups against each other on our balance scale. If they weigh the same, then we will know that the fake coin is in the 3rd group. If one of the groups is lighter, we know the fake is in that group. Either way, we will have a group of 3 coins, and we will know the fake coin is among them. Then we will repeat the solution of Problem 1. And we will be done with a total of only 2 weighings.

## Problem 3

Seven coins are arranged in a circle. It is known that four of them going in a row are fake and that every fake coin is lighter than a real one. Explain how to find two fake coins in a single weighing on a balance scale. (All fake coins weigh the same.)

Solution: Let us pick the top coin and the 3rd coin from it counting clockwise. We will weigh them. If the weigh the same, then they are along with the 2 coins between them are fake. Remember, there must be a row of 4 fake coins, and a row of 3 real coins. If 2 coins separated by 2 more coins happen to weigh the same then they must be the endpoints
 of the chain of 4 fakes. If they happen to weigh differently, then the lighter one must certainly be a fake. But we need to find an additional fake coin to have a solution. We will then take the coin next to the fake one in the direction opposite to the heavier coin. This coin must be a fake one. We know this, because if it were a real one, the heavier coin we just measured cannot possibly be real. Because there cannot be a row of 4 real coins.

## Today, we will be playing the game of "show me your sister".

There are 15 students in the class and 2 pairs of siblings among them. How can we show that they are siblings? We can simply draw a line connecting the 2 students who are siblings.

How do we show 2 sisters in a family? We will draw a red arrow from one sister to another. So there will be 2 arrows in total, as both girls are each other's sisters. How about a family with a brother and a sister? There will be one arrow from the brother to the sister, as he is "showing his sister". But the sister cannot point an arrow to her brother, since he isn't her sister, but a brother instead.


Now we have 15 kids in the yard playing the game "Show me your sister". Can you tell who is who on the picture? If there are 2 dots both pointing arrows towards each other, who can they be? They must be each other's sisters. How about a dot pointing to another dot? Probably a brother pointing to his sister. If there are 2 dots pointing to a third dot? Must be 2 brothers pointing to their common sister.
3. Show me your sister or your brother. Let us now draw a better picture. We will point a red arrow towards sisters and a blue arrow towards brothers. How do we draw a family that has 3 siblings and all
 of them girls? We will draw red arrows pointing from every sister to every other sister because everyone is everyone else's sister. We will then have a total of 6 red arrows. How about a family of 2 sisters and a brother? To each of the sisters, everyone else will point a red arrow, and the 2 sisters will point a blue arrow towards the brother. This will give us a total of 4 red +2 blue $=6$ arrows.


Now let us try to count how many arrows in total we will have in families of different sizes. How many arrows will there be if we have 2 siblings? 3 siblings? 4 siblings? 100 siblings? If I am one of the 3 siblings in a family, how many arrows will point towards me? 2 arrows from my 2 siblings will point towards me. Now how many children in total will have arrows pointed towards them in the family? 3 children, because they are all siblings of everyone else. That means in this family there will be 2 arrows pointing towards each of the 3 children for a total of 6 arrows. In a family with 5 siblings, there will be 4 arrows pointing towards each of the 5 children, making a total of 20 arrows. How about a family of 100 siblings? 99 arrows pointing towards each of the 100 children, or 99x100=9900 arrows in total.


Let us play a game now! Two players will play on the board, where we have 9 dots for a class of 9 students. The rules are simple: the players take turns drawing an arrow of the right color from one of the students to his/her sibling. When drawing an arrow to a dot without any arrows already, the player can choose to make it a brother or a sister. The first player who is unable to draw an arrow loses the game.
Remember: we cannot have more than 3 siblings in any family; all arrows pointing to a dot should have the same color.


