## Mazes, dragons and a Game of Sprouts.

## We start with some Heads and Legs problems.

Problem 1. There were 10 bicycles with 27 wheels in total. All bicycles had 2 wheels or 3 wheels. How many twowheeled bicycles and how many threewheeled bicycles were there?
Solution: Let us see what happens if all bicycles have 2 wheels. We would have 20 wheels among our 10 bicycles. But we

need 27 in total! That means 7 wheels will be missing if all bicycles had 2 wheels. How many of our bicycles should then have 3 wheels? A 3 -wheel bicycle has 1 more wheel than a 2 -wheel bicycle. That means 7 of our 2 -wheel bicycles need to become 3 -wheeled.
So our answer is 73 -wheel bicycles and 32 -wheel bicycles!
Problem 2. On a farm near Berlin, there were 21 eggs. Some of them hatched ducklings, and the rest hatched lizards. In the end, there were the same number of duckling legs as lizard legs. How many ducklings were there and how many lizards?
Solution: Something tells me that ducklings have 2 legs each, and lizards have 4 legs each. How many chickens would have the same number of legs as a single lizard? 2 chickens equals 1 lizard. That is total of 3 animals to keep the number of chicken legs and lizard legs equal. But how many of these 3 animal combinations ( 2 chickens and 1 lizard) do we have? 21 divided by 3 is 7 . So we must have 7 lizards and $2^{*} 7=14$ chickens.


Problem 3. On a pirate ship, there were a few cats, a few sailors, a cook and one-legged captain. All of them, taken together, have 15 heads and 41 legs. How many cats were there on the ship?

Solution: Let us first get rid of the known things: a cook gives us 1 head and 2 legs, captain gives us 1 head and 1 leg. We will throw them to the ocean. Now we are left with a few cats and a few soldiers. In total they give us 13 heads and 38 legs. Now let us see what happens if all 13 heads were sailor heads: we would have only 26 legs. But we need 38 legs, which means we need to have the 38-26=12 addition legs come from the cats. How many additional legs does replacing a sailor with a cat give us? 4-2=2 additional legs! Now to take care of the 12 additional legs needed, we would replace $12 / 2=6$ sailors with cats.

Then we would have $13-6=7$ sailors and 6 cats.

## Can arrangements of dots and strings turn into each other?

Let us have a set of dots that are connected somehow among themselves by strings. For example, we have a set of five dots where each dots is connected to two other dots. What shape can we build out of it without cutting the strings? We are allowed to move the dots and the strings will follow them.
We can definitely make a pentagon arrangement out of it.
Now what can we turn it into? Can we make a star out of it? How about a trapezoid? How about a cross arrangement?


Is there a way to know which arrangement can or can not be built using our closed chain of 5 dots? Let us start counting how many other dots each dot is attached to. In our closed chain, we have ( $2,2,2,2,2$ ) neighbors. How about a star? Also ( $2,2,2,2,2$ ) neighbors. So maybe it is possible to build the star? Now the cross arrangement would have (4, 1, 1, 1, 1) neighbors. And since we are not allowed to break the strings from the dots and reconnect to other dots, we definitely cannot make the cross arrangement.


Two arrangements are considered the same if one of them can be obtained from the other by moving the dots and bending the strings, without breaking or re-attaching dots and strings.




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## Now we go for treasure hunt!

We have five maze arrangements on the worksheet, each of which has a treasure and a dragon. There is one arrangement of dots and strings on the board. We need to find which arrangement from worksheet is on the board, and mark on the board the entrance to the maze, the rooms with dragon and with treasure.





## Finally we played the Game of Sprout.

The game was invented in 1967 by two mathematicians John H. Conway and Michael S. Paterson. All you really need is paper and a pencil. The game is played by two players, starting with a few spots drawn on a sheet of paper.

Players take turns, where each turn consists of drawing a line between two existing spots (or from a spot to itself) and adding a new spot somewhere along the line.


The players are constrained by the following rules.
The line may be straight or curved, but must not touch or cross itself or any other line.
No spot may have more than three lines attached to it. For the purposes of this rule, a line from the spot to itself counts as two attached lines and new spots are counted as having two lines already attached to them.

The idea is to make it impossible for the other player to draw a line.
So the last person to draw a line is the winner.
Can you figure out the strategy for winning this game?


See you next week!

