Rings and chains.

Patterns. We started our lesson with set of problems related to patterns. One of the most useful strategies in math is problem solving by finding a pattern. Searching for visual patterns can be as simple as identifying the change from one item in a sequence to the next. We looked at problems that includes number patterns, changes in color and shapes, rotation, and more. You can find some of the problems we worked on below and more problems here.



Sophie is arranging black and white marbles following the pattern shown in the picture.


Sylvia drew figures with hexagons like in the picture. How many hexagons will the fifth figure contain, if she continues with this pattern?


We make a sequence of figures with tiles. The first four figures have 1, 4, 7 and 10 tiles, respectively.
How many tiles will the fifth figure have?

$\square \square \Rightarrow \triangle \Rightarrow$
$\Delta \Rightarrow \square \Rightarrow \square$
Objects on top of each other: first - see it. Let us play the game: close your eyes and we will put 3-4 geometrical shapes on top of each other. Now you can open your eyes. Can you tell us which object did we put first? Second? Last? If we can see the full shape what does it mean? Now we know how it will look like. Can we color the picture using the scheme that shows us in what order we put shapes?


Let us now play with rings and chains. Rings on top of each other and connected

- Let's draw 2 overlapping rings, and color then as sitting on top of each other
- Let us now color them to be linked together
- How about 3 rings that are chained into a link? Or 3 rings that are not linked, just sitting on top of each other?



## Problems on disconnecting and connecting chains

- Let us take a chain of 3 links. How many links do I need to break to disconnect the entire chain into 3 individual links? 2 - by breaking the leftmost and rightmost links? That works! But is there a "better" solution? What if we break just one link - the middle one? Does the chain break into separate links?
- If I have a chain of 7 links, by breaking how many links can I turn it into 5 separate pieces? If we break the 1 st and last links, we only get 3 separate pieces. Can we do better? What if we break the $2 n d$ as well as $2 n d$ last links? We then get 1st link alone, 2nd broken link alone, links 3 through 5 together in one piece, 6th (2nd last) broken link alone and the 7th, last link alone.


## Now we are ready for a more difficult problem:

A jeweler is asked to join five small 3-link chains into a single large 15-link chain. In order to join two closed links, one of the links needs to be cut, placed onto the other link, and then closed.
The clever jeweler finds a creative way to make the 15 -link chain while cutting as few links as possible.
How many links does he cut and what does he do?


## Drawing a chain in different ways.

I think drawing 2 nested circles just to show a single chain link is completely wasteful. Let us try to find a very simple way to draw any chain of links. If there are 2 separate links sitting away from each other, how can we show them on a board in the easiest way possible? 2 separate dots! OK, that works! How about 2 links that are connected to each other? How would we show that? By drawing a line between them. How about a long chain of 6 links? We can just draw 6 dots and connect them using a single line! Now let us try some more chains that are linked in very complicated ways.


Now let us see if any of the chains we see are actually the same things. Is chain 1 the same as chain 2 ? No, chain 1 is a simple stretch of 6 links, and chain 2 has a more interesting shape. How about chains 2 and 5, are they the same? Let's make them out of our fuzzy stick pipe-cleaners and compare by turning around and playing with it. They both look like a chain of 5 links, with the 6th link connected to the 2nd link in the long chain.

