## Divide-and-compare.

## How can we divide a square into 2 equal pieces?

Let us try to divide a square into 2 identical pieces using a single straight line. How hard can it be? And how many different ways are there anyways?

- We have a solution of dividing with a single line going right through the middle from top to bottom. This is exactly the same solution as a line through the middle from left to right! Just turn the square once.
- We have another solution of dividing with a single line along the diagonal, from top left corner to bottom right corner. Any more solutions left?
- How about a line going from left to right but not from a corner and not from the middle of the left side. Instead we will draw from a random point on the top half of the left side, through the center of the square and to a point on the bottom half of the right side. Just like in the 3rd square on the picture below.
- Now let us try to find another solution! Is it even possible to find a 4th solution? What if we noticed that our line is always going through the center of the square? Then we can just rotate
 our line going through the center a little bit and get a completely different solution! Just like in the 4th square on the picture below.


## How about dividing into 4 equal pieces?

- Let us simply cut the square into 4 smaller squares! Does that work?
- Does cutting the square into 4 triangles also work?
- Can it be cut into 4 equal thin strips?
- How about 4 equal, but long triangles like in the 4th square on the picture below?
But when we divide the square into pieces, how can we make sure that the pieces are all equal? Is there an easy
 way to show that shapes are the same? I think we can simply turn the pieces around and put them on top of each other. What do you think?



## How many squares make a square?

What is the smallest square we can make using small identical square-shaped pieces? Just a single square! How many do we need for the next larger square?

- Would 2 squares make a larger square? No! Only 4 squares will make a larger square!
- The next larger square can be made using 9 squares, because 9 is 3 times 3 !
- How about the next one?
- If Lena kept building squares of increasing sizes, how many small squares would she need on her 10th shape? 10 times 10 ! Which is 100 squares!

Can we divide a 4-by-4 grid into equal pieces?


Let us now divide a 4-by-4 square grid into 2 equal pieces

by cutting only along the sides of the small squares! I think I can divide a 4-by-4 square grid by cutting it right down the middle from top to bottom. Who can come up with other ways? We may cut it like this:

1111
1122
1122
2222

How about:
1112
1222
1112
1222

Let us now see if we can divide some very strange shapes!
Can we divide into 2 identical pieces the shape to the right? Or how about a shape right below it? Is dividing shapes into 3 equal pieces any more difficult?

Let us try and see if we can tell if any given shape can be divided into 2 pieces. Can we divide a shape made out of odd number of squares into 2 equal pieces?


Can we divide shapes into equal pieces if each piece needs one fruit?


Let's divide the green 4-by-5 rectangle below into 2 equal shapes made of small square so that each piece has a single red apple in it. Or we can divide the thick red letter $T$ into 4 equal pieces so that each one of them has a single yellow apple in it.

|  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  | $\ddots$ |  |
|  |  |  |  |  |
|  |  |  |  |  |



How about dividing a 4-by-8 rectangle into 8 equal pieces containing an apple each? Or dividing a 6-by-6 square into 4 equal pieces containing an apple and a plum each?


See you next week!

