## Mirror Symmetry

Let us start with a very simple task: Lena will put on the board some arrangement of shapes and draw a line for a mirror. And the task will be to make the "mirror image" of Lena's arrangement on the other side of the mirror. If there is a hexagon touching the mirror, there has to be another hexagon on the other side of the mirror also touching it. If there are 2 squares attached to the hexagon, then there needs to also be 2 squares attached to the other hexagon, in the exactly opposite to the mirror
 sides.

I have 4 circles on the whiteboard. They are arranged like this: red - in top left corner, yellow in bottom right corner, and 2 blue circles - in top right and bottom left corners. How can I then get an image of a single blue circle using a straight mirror? We can simply put our mirror through the middle of one blue circle so that the mirror is facing away from all other circles. Then the mirror will reflect
 one half of this circle and along with the real half, there will be a full blue circle. And all other circles will be hidden because they are behind the mirror. They won't be visible to us and the mirror won't reflect them to us either. How about making 3 blue circles and 2 yellow circles? We will have to put the mirror so that the yellow circle, one full blue circle and one half of the other blue circle are on the mirror's one side so that they all get reflected and give us double of everything. This will make 2 full blue circles, 2 half blue circles (together making 1 more full blue circle), and 2 full yellow circles.

We have a large triangle with smaller triangles inside. There are green, blue, red and orange triangles, one of each, inside this large triangle. Now our task is to make the images of the 4 triangles given on the board (also large, with triangles inside, but with some colors repeating). How can we make a large triangle with a small red triangle at the top, and 2 small blue triangles at the bottom? What if we draw the "line of symmetry" for this image and look at the halves? It will go from top to bottom of the large triangle, and pass through the middle of the red and orange triangles, and separate the blue triangles. That means our mirror has to go through the middle of the red and orange triangles as well, and look towards the blue triangle.

Now we have quite an entertaining problem: We have a picture of a real girl inside the yellow frame. And we know that the images numbered 1-5 were made using a mirror from the real girl's picture. We just need to figure out how we have to put the mirror to get these strange images. Let's look at image 1: There are 2 braids, 2 bow-ties, 2 eyes, correctly drawn nose and mouth. So the line must be going through the middle of the girl from top to bottom. But which way should the mirror be pointed?

(3)


(4)

(2)
 (5)
 We don't see her tongue, and the real girl is sticking her tongue out on the left side. Then the mirror must be pointed the other way: to the right.


We have an even more interesting problem here: Lena's son left a big mess of toys on his bedroom floor. And it turns out, during the night, some monsters appear when looked in the mirror: There are images of some very strange looking monsters, and you will have to figure out which way we need to put the mirror on the floor full of
 toys to see each of the monsters.


Our final task is a difficult one: we need to find an image that cannot be made from our bird picture using a single straight mirror. For each of the numbered images 1-8, let us try to find where we need to put the mirror on our bird inside the blue frame to get it. Our task becomes somewhat easier if we try drawing the line of symmetry, the line dividing the image in half that are exact opposite of each other. Can we draw a line of symmetry through image 6?

