

Show me your sister.

Problem 1

There is a balance scale with ten boxes on left side and six boxes on the right side. All boxes have the same exact weight. Lena's dog Marta is hiding in one of the boxes and the pans are balanced. Where is the box with Marta in it? On the left side of the scale or the right side?

Problem 2

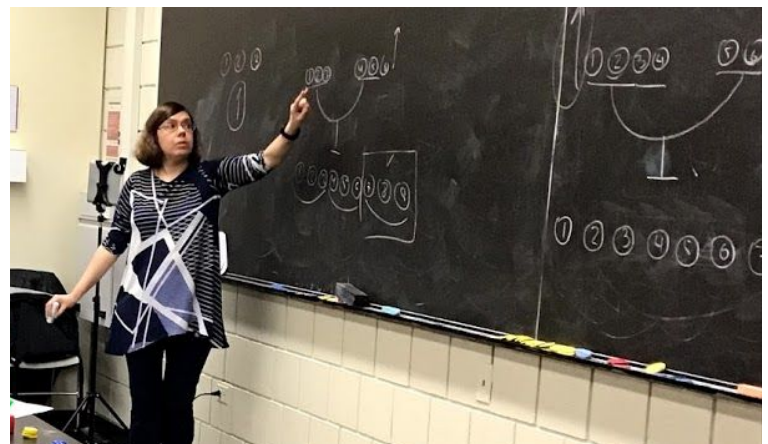
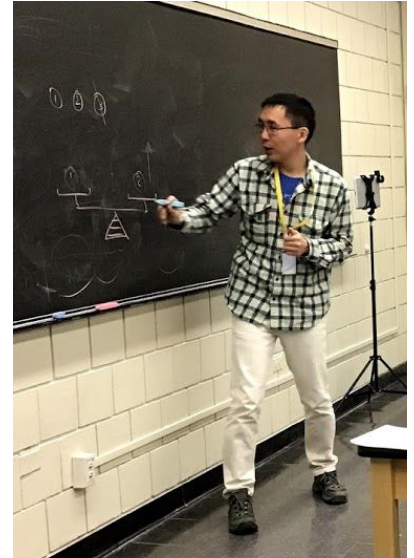
There are three coins, one of them is fake (lighter than a real ones). How many weighings do we need to figure out which coin is fake?

Solution: Let's weigh any two first. If they are the same, the third coin must be a fake. If one of them is lighter, then that one is the fake. It took us only 1 weighing to find the fake one!

Problem 3

There are 9 coins, one of them is fake (lighter than a real ones). How can we figure out for two weighings which coin is fake?

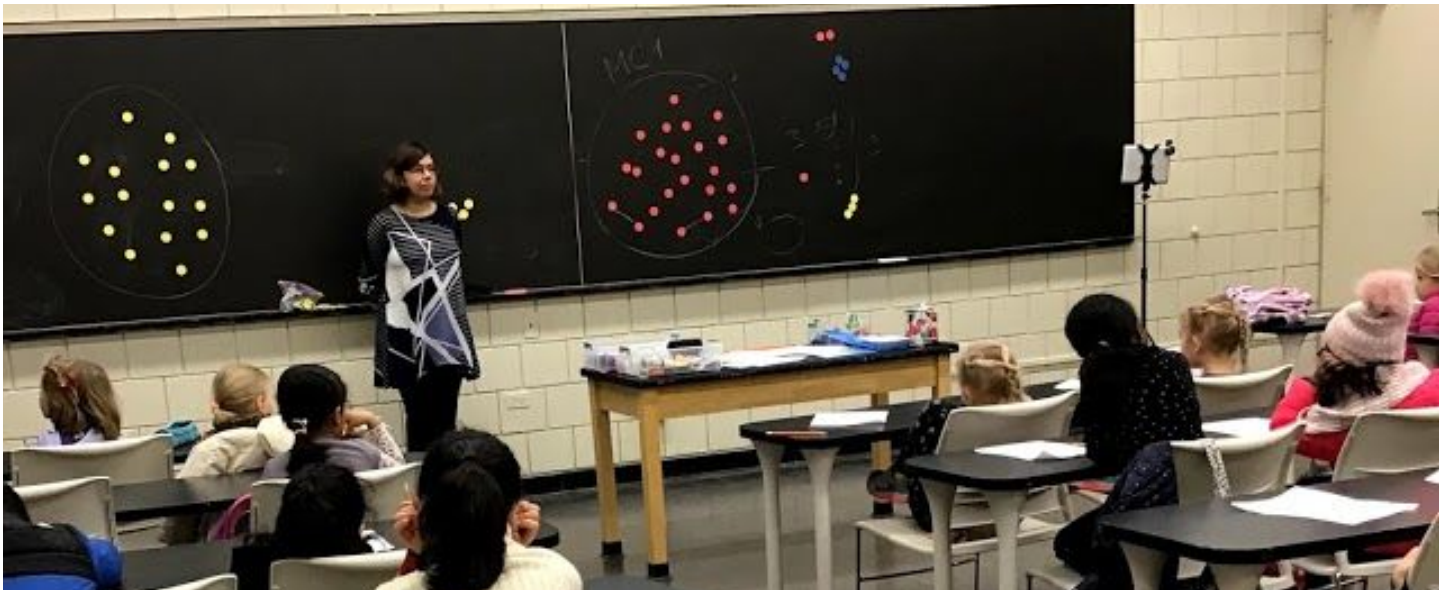
Solution: We split the coin into 3 groups of 3 coins each. We will then weigh 2 of the groups against each other on our balance scale. If they weigh the same, then we will know that the fake coin is in the 3rd group. If one of the groups is lighter, we know the fake is in that group. Either way, we will have a group of 3 coins, and we will know the fake coin is among them. Then we will repeat the solution of Problem 1. And we will be done with a total of only 2 weighings.



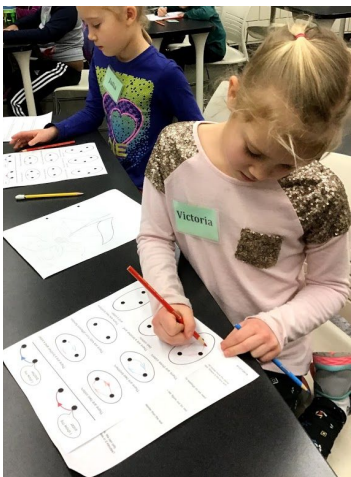
Today, we will be playing the game of "show me your sister".

There are 15 students in the class and 2 pairs of siblings among them. How can we show that they are siblings? We can simply draw a line connecting the 2 students who are siblings.

How do we show 2 sisters in a family? We will draw a red arrow from one sister to another. So there will be 2 arrows in total, as both girls are each other's sisters. How about a family with a brother and a sister? There will be one arrow from the brother to the sister, as he is "showing his sister". But the sister cannot point an arrow to her brother, since he isn't her sister, but a brother instead.



Now we have 15 kids in the yard playing the game "Show me your sister". Can you tell who is who on the picture? If there are 2 dots both pointing arrows towards each other, who can they be? They must be each other's sisters. How about a dot pointing to another dot? Probably a brother pointing to his sister. If there are 2 dots pointing to a third dot? Must be 2 brothers pointing to their common sister.



3. **Show me your sister or your brother.** Let us now draw a better picture. We will point a red arrow towards sisters and a blue arrow towards brothers. How do we draw a family that has 3 siblings and all of them girls? We will draw red arrows pointing from every sister to every other sister because everyone is everyone else's sister. We will then have a total of 6 red arrows. How about a family of 2 sisters and a brother? To each of the sisters, everyone else will point a red arrow, and the 2 sisters will point a blue arrow towards the brother. This will give us a total of $4 \text{ red} + 2 \text{ blue} = 6 \text{ arrows}$.

