Math Club 1. Lesson # 14 **January 20, 2019**

Mazes and Dragons.

We start with some Heads and Legs problems.

Problem 1. There are rabbits and chickens in the basket. How many rabbits are in the basket if there are 3 heads, and 10 paws altogether?

Solution: Something tells me that rabbits have 4 paws each, and chickens have 2 paws each. But each rabbit and each chicken have only 1 head. Let us see what happens if all 3 of our animals were chickens? There would be 3 animals with 2 paws each giving us 6 paws. But we need 10 paws, correct? Where do we get the missing 10-6=4 paws? We change some of our chickens to rabbits. How many chickens need to turn into rabbits? Every time we turn a chicken into a rabbit, we will add 4-2=2 legs, because a rabbit has 2 more legs than a chicken does. To get 4 more legs, we then need to turn 2 chickens into rabbits, getting 2+2=4 more legs in return.

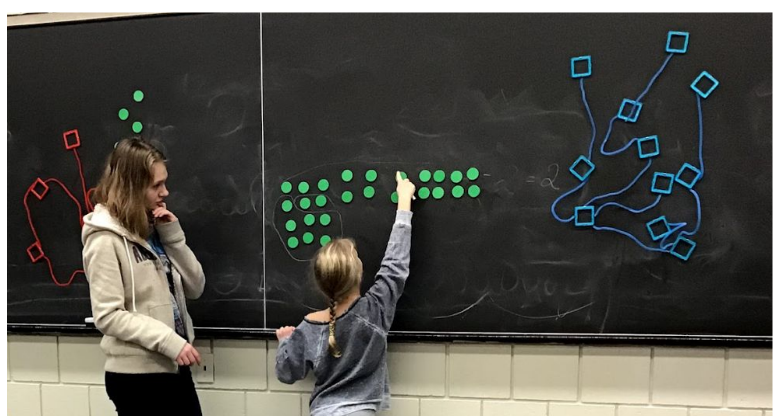
So our answer is 2 rabbits!

Problem 2. There were 5 bicycles with 12 wheels in total. All bicycles had 2 wheels or 3 wheels. How many two-wheeled bicycles and how many three-wheeled bicycles were there?

Solution: Let us see what happens if all bicycles have 2 wheels. We would have 10 wheels among our 5 bicycles. But we need 12 in total! That means 2 wheels will be missing if all bicycles had 2 wheels. How many of our bicycles should then have 3 wheels? A 3-wheel bicycle has 1 more wheel than a 2-wheel bicycle. That means 2 of our 2-wheel bicycles need to become 3-wheeled.

So our answer are 2 3-wheel bicycles and 3 2-wheel bicycles!

Problem 6



The king had several dragons in his basement: some 2-headed and some 7-headed dragons. Each morning he counted his dragons by their head to make sure everyone was fine. This morning he counted 26 heads. How many dragons does the king have?

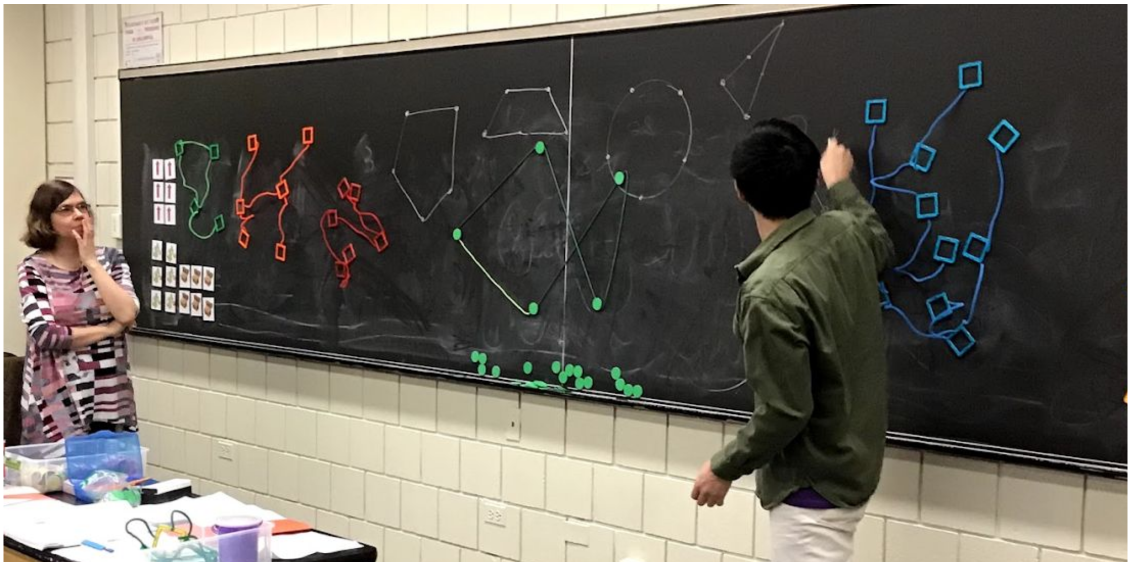
Solution: Let's try to imagine the number of heads on 2-headed dragons and separately on 7-headed dragons. On 2-headed dragons there will be some even number of heads. If we had 1 7-headed dragon, how many heads will be left for 2-headed dragons? 26-7=19 heads will be shared between 2-headed dragons, but that is not possible, because we need an even number of heads on 2-headed dragons. How about 2 7-headed dragons? We would have 7+7=14 heads for them, and 26-14=12 heads for the 2-headed dragons. That means we can have 6 2-headed dragons, because 6+6=12. 2 7-headed dragons and 6 2-headed dragons together give us 2+6=8 dragons!

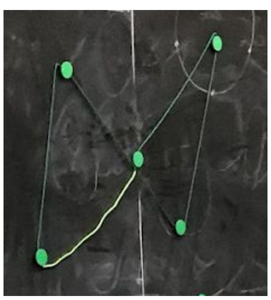
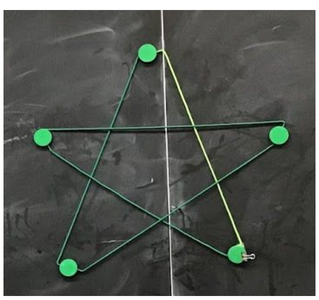
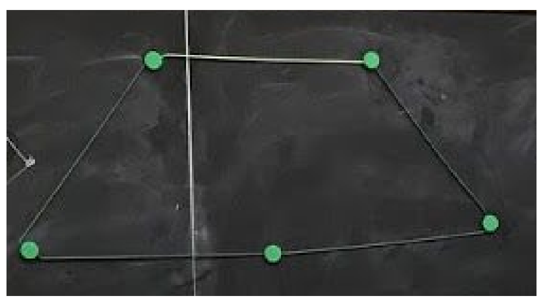
So our answer is: 8 dragons in total.

Can arrangements of dots and strings turn into each other?

Let us have a set of dots that are connected somehow among themselves by strings. For example, we have a set of five dots where each dots is connected to two other dots. What shape can we build out of it without cutting the strings? We are allowed to move the dots and the strings will follow them. We can definitely make a pentagon arrangement out of it.

Now what can we turn it into? Can we make a star out of it? How about a trapezoid? How about a cross arrangement?



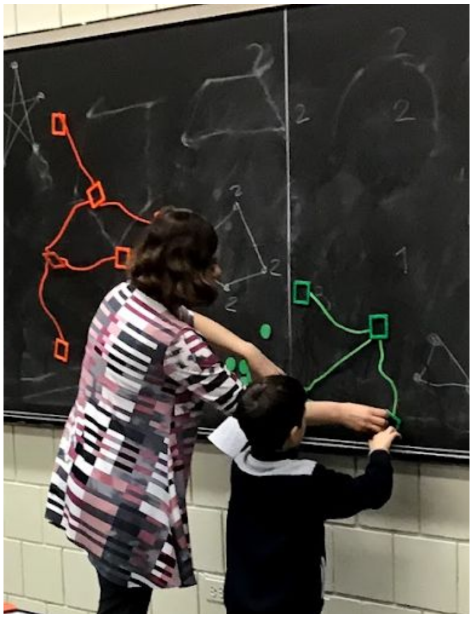
Is there a way to know which arrangement can or can not be built using our closed chain of 5 dots?

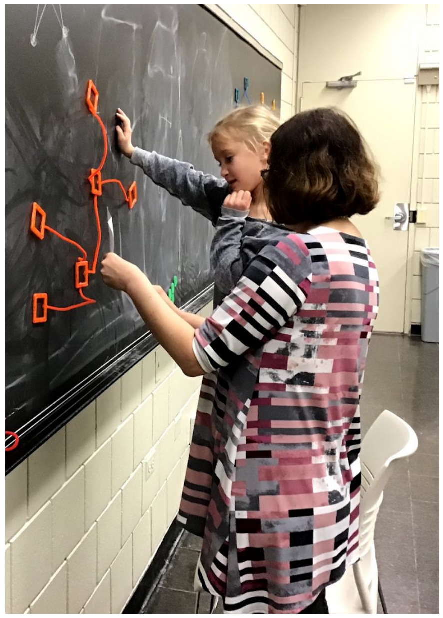
Let us start counting how many other dots each dot is attached to. In our closed chain, we have (2, 2, 2, 2, 2) neighbors. How about a star? Also (2, 2, 2, 2, 2) neighbors.

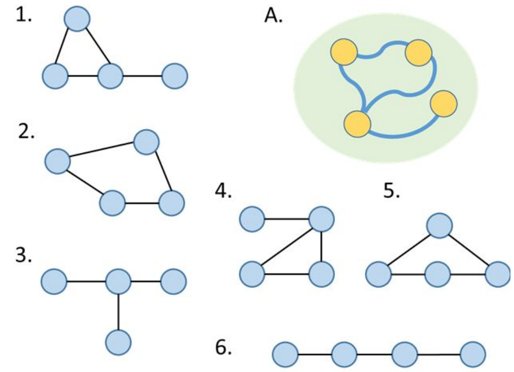
So maybe it is possible to build the star?

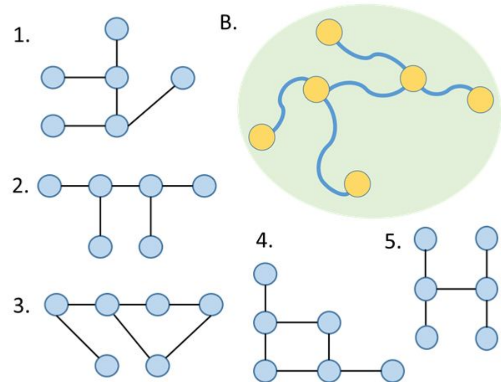
Now the cross arrangement would have (4, 1, 1, 1, 1) neighbors. And since we are not allowed to break the strings from the dots and reconnect to other dots, we definitely cannot make the cross arrangement.

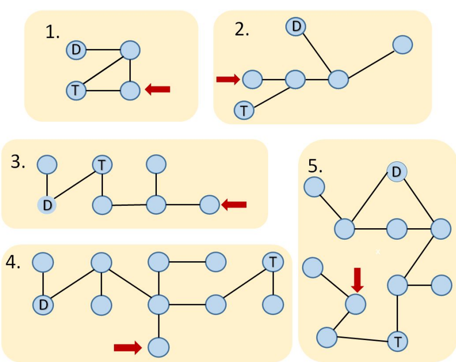
Two arrangements are considered the same if one of them can be obtained from the other by moving the dots and bending the strings, without breaking or re-attaching dots and strings.











Now we go for treasure hunt!

We have five maze arrangements on the worksheet, each of which has a treasure and a dragon. There are also five arrangement of dots and strings on the board. We need to find which maze from the worksheet is corresponds to the each arrangement on the board, and mark on the board the entrance to the maze, the rooms with dragon and with treasure.