1. Homework

1. Let A, B, C be on a circle centered at O such that $\angle AOB \cong \angle BOC \cong \angle COA$. Prove that $\triangle ABC$ is an equilateral triangle.

2. Let A, B, C, D be points on circle ω that form a quadrilateral. Prove that $m\angle ABC + m\angle ADC$. We call such a quadrilateral a cyclic quadrilateral: it is inscribed in a circle.

3. Let $\triangle ABC$ be an isosceles triangle with base $\overline{AC}$ whose perpendicular bisectors meet at point O. Prove that $\angle BOA \cong \angle BOC$.

4. Given points A, B, what is the locus of points C such that $m\angle ACB = x^\circ$ for some number x?

5. Suppose $\triangle ABC$ and $\triangle DAB$ are isosceles triangles with bases $\overline{AC}$ and $\overline{BD}$, respectively, such that $\overline{BD}$ is an altitude of $\triangle ABC$. Prove that $CD = AB$.

6. Let λ be a circle whose center is inside circle ω such that λ, ω intersect at points P, Q. Let $\overline{AB}$ be a diameter of λ such that A, B are also inside ω; then, let $\overleftrightarrow{PA}$ and $\overleftrightarrow{PB}$ intersect ω at points X, Y respectively. Prove that $\overline{XY}$ is a diameter of ω.

7. Let $\triangle ABC$ and $\triangle ADC$ be right triangles such that A, B, C, D lie on a circle and $m\angle BAC = 90^\circ$. Prove that $ABCD$ is a rectangle.

8. Let circle ω have radius r with center O and λ have radius s with center P, with $r \neq s$, $O \neq P$. Let circle ω' have radius r with center P, and λ' have radius s with center O (i.e., the circles with the same radii but opposite centers). Let ω, λ intersect at points A, B and ω', λ' intersect at points A', B'; what sort of quadrilateral is $ABB'A'$? Let circles ω, ω' intersect at points X, X'; what sort of quadrilateral is $ABXX'$?

9. Given a line segment $\overline{AB}$ such that $AB = 1$, construct C on $\overrightarrow{AB}$ such that:

(a) $AC = \frac{1}{4}$
(b) $AC = \frac{1}{3}$
(c) $AC = \frac{1}{6}$
(d) $AC = \frac{1}{5}$
(e) $AC = \sqrt{2}$
(f) $AC = \sqrt{3}$
(g) $AC = \sqrt{5}$
(h) $AC = \sqrt{7}$

10. Let $ABCD$ and $ABEF$ be parallelograms such that E, F are on the line $\overleftrightarrow{CD}$; let the diagonals $\overline{AC}$, $\overline{BD}$ intersect at M and $\overline{AE}$, $\overline{BF}$ intersect at N. Prove that $\overline{MN} \parallel \overline{AB}$.