

MATH 8
ASSIGNMENT 1: COMBINATORICS REVIEW
SEPTEMBER 16, 2018

WELCOME TO THE NEW SEMESTER AT SCHOOLNOVA!!

This Fall, we plan to study the following topics:

- Review of combinatorics. Binomial formula.
- Logic and proofs

We will try to do much of the homework in class so that you do not need to spend too much time on it at home. As usual, all HW assignments and other information will be posted online at <http://www.schoolnova.org>

I ask that each student bring a notebook (preferably quad ruled), pencils and a folder or binder to keep old assignments — you will need them!

We also plan to participate in two math competitions: Math Kangaroo and American Math Contests (AMC). Math Kangaroo is an international math competition for all ages; you can find more information on their web site at <http://www.mathkangaroo.org> . The contest is in March. Details of the registration will follow.

AMC (<http://amc.maa.org/>) is the “official” American Math Olympiad: it is the first level of the competition that eventually leads to the selection of US team for International Math Olympiad. AMC 8 is intended for students in grades 8 and below. This year, AMC 8 will be given on Tuesday, November 13, 2018. You do not have to register individually – just let me know if you are interested.

If you have any questions, please contact us by email: nine@schoolnova.org, kirillov@schoolnova.org

COMBINATORICS: A REVIEW!

1. A club consisting of 25 people need to choose the president, vice-president, and treasurer. In how many ways can they do this?
2. In a meeting of 25 people, every one of them shakes hands once with every other. How many handshakes was it altogether?
3. There is a round table seating 8. How many ways there are for 8 people to choose their seats at the table? What if we do not distinguish between two seatings which only differ by rotating the table?
4. How many words one can get by permuting letters of the word “tiger”? of the word “rabbit”? of the word “common” of the word “Mississippi”?
5. If we draw 3 cards out of the deck, what are the chances that
 - They will all be all spades
 - They will be all aces
 - That they will be ace of spades, queen of spades, and king of spades, in this order
 - That they will be queen of spades, ace of spades, and king of spades, in this order
 - * That they will be ace, queen, and king of spades, in some order
6. How many different paths are there on 5×5 chessboard connecting the lower left corner with the upper right corner? What about 6×6 ? The path should always be going to the right or up, never to the left or down.

7. How many “words” of length 5 one can write using only letters U and R, namely 3 Us and 2 Rs? What if you have 5 Us and 3 Rs? [Hint: it is related to the previous problem – each such “word” can describe a path on the chessboard, U for up and R for right...]
8. A drunkard is walking along a road from the pub to his house, which is located 1 mile north of the pub. Every step he makes can be either to the north, taking him closer to home, or to the south, back to the pub – and it is completely random: every step with can be north of south, with equal chances. What is the probability that after 10 steps, he will end up
 - (a) 10 steps north from the starting position
 - (b) 10 steps south
 - (c) 8 steps north
 - (d) will come back to the starting position
- *9. You have 10 books which you want to put on 2 bookshelves. How many ways are there to do it (order on each bookshelf matters)?