MATH 8 ASSIGNMENT 11: LOGIC REVIEW DEC 9, 2018

NOTATION REMINDER

Logical operations: \lor or; \land and; \neg not; \Longrightarrow implies (same as "if...then..."); Quantifiers: $\forall x \in A : \ldots$ for any x in set A, ...

 $\exists x \in A : \dots$ there exists an x in set A such that ...

Common notation for sets:

 \mathbb{R} : set of real numbers

 \mathbb{N} : set of poistive integers $(1,2,\ldots)$

 \mathbb{Z} : set of integers (including zero and negatives)

Problems

When doing these problems, you can use the following standard facts about sets \mathbb{R} and \mathbb{Z} :

- Laws for addition and multiplication: commutativity, associativity, distributivity
- Rules for 0 and 1: 0 + x = x, $1 \cdot x = x$, $0 \cdot x = 0$.
- Rules for negatives: for any x, there is a unique number -x such that x + (-x) = 0, and -(-x) = x.
- (For set \mathbb{R} only): any nonzero $x \in \mathbb{R}$ has an inverse: there exists a unique number x^{-1} such that $x \cdot x^{-1} = 1$.
- **1.** Given statements A and B, if I know that $\neg(A \land B)$ is true and I know that A is true, what can I conclude about B?
- **2.** An integer number a is called even if $\exists n \in \mathbb{Z} : a = 2n$. A number a is called odd if $\exists n \in \mathbb{Z} : a = 2n + 1$. You can use without proof the fact that every integer is either even or odd, but not both.
 - (a) Prove that if a number is even, then its square is also even
 - (b) Prove that if integers a, b are even, then a + b is also even.
 - (c) Prove that an integer number a is odd if and only if a^2 is odd
 - (d) Show that if mn is even, then m is even or n is even.
- **3.** For integer numbers a, b, we say that a divides b, or that b is divisible by a (notation: a|b) is there exists an integer n such that b = na.
 - (a) Prove that if a|b and a|c, then a|(b+c).
 - (b) Prove that if a|b but a doesn't divide c, then a doesn't divide b + c.
- 4. Prove that a positive integer a is odd if and only if it can be expressed as a difference of consecutive squares. (Here, *consecutive squares* means the squares of two positive integers k, k+1).
- 5. It is known that integer 1 is not divisible by any positive integer other than 1. Use it to prove:
 - (a) If integer n is even, then n + 1 is not divisible by 2.
 - (b) If integer n is divisible by 5, then n + 1 is not divisible by 5.

- (c) If n + 1 is divisible by 5, then n is not divisible by 5.
- (d) For any integer n > 1, x and n + 1 have no common factors other than 1. (A common factor is a positive integer k that divides both of the integers in question.)
- **6.** Prove or disprove the following statement: $\forall p \in \mathbb{Z} \ (p \text{ is prime} \implies p+1 \text{ is not a power of } 2)$
- (a) Prove that if x, y ∈ R are such that xy = 0, then x = 0 ∨ y = 0. [Hint: x has an inverse.]
 (b) Prove (using nothing but the basic facts about reals given above) that x² 5x + 6 = 0 if and only if x = 2 or x = 3. Here x is a real number.
- 8. Recall that the product of any two positive real numbers is positive. Given real numbers x, y, z, such that y > z and xy < xz, prove that x < 0.
- **9.** The standard card deck has 52 cards (4 suits, each with 13 values, from 6 to ace). You have drawn 2 cards, which happened to be king of spades and 9 of hearts.

You now draw 3 more cards. What is the probability that your 5-card hand will contain the following combinations:

- (a) Four of a kind: four cards of the same value
- (b) Three of a kind: three card of the same value (but not 4 cards of the same value)
- (c) Two kings and two nines (but no three of a kind)
- (d) Three kings and two nines.