# MATH 8 ASSIGNMENT 11: LOGIC REVIEW <br> DEC 9, 2018 

## Notation Reminder

Logical operations: $\vee$ or; $\wedge$ and; $\neg$ not; $\Longrightarrow$ implies (same as "if...then...");
Quantifiers: $\forall x \in A: \ldots$ for any $x$ in set $A, \ldots$
$\exists x \in A: \ldots \quad$ there exists an $x$ in set $A$ such that ...
Common notation for sets:
$\mathbb{R}$ : set of real numbers
$\mathbb{N}$ : set of poistive integers $(1,2, \ldots)$
$\mathbb{Z}$ : set of integers (including zero and negatives)

## Problems

When doing these problems, you can use the following standard facts about sets $\mathbb{R}$ and $\mathbb{Z}$ :

- Laws for addition and multiplication: commutativity, associativity, distributivity
- Rules for 0 and 1: $0+x=x, 1 \cdot x=x, 0 \cdot x=0$.
- Rules for negatives: for any $x$, there is a unique number $-x$ such that $x+(-x)=0$, and $-(-x)=x$.
- (For set $\mathbb{R}$ only): any nonzero $x \in \mathbb{R}$ has an inverse: there exists a unique number $x^{-1}$ such that $x \cdot x^{-1}=1$.

1. Given statements $A$ and $B$, if I know that $\neg(A \wedge B)$ is true and I know that $A$ is true, what can I conclude about $B$ ?
2. An integer number $a$ is called even if $\exists n \in \mathbb{Z}: a=2 n$. A number $a$ is called odd if $\exists n \in \mathbb{Z}: a=2 n+1$. You can use without proof the fact that every integer is either even or odd, but not both.
(a) Prove that if a number is even, then its square is also even
(b) Prove that if integers $a, b$ are even, then $a+b$ is also even.
(c) Prove that an integer number $a$ is odd if and only if $a^{2}$ is odd
(d) Show that if $m n$ is even, then $m$ is even or $n$ is even.
3. For integer numbers $a, b$, we say that $a$ divides $b$, or that $b$ is divisible by $a$ (notation: $a \mid b$ ) is there exists an integer $n$ such that $b=n a$.
(a) Prove that if $a \mid b$ and $a \mid c$, then $a \mid(b+c)$.
(b) Prove that if $a \mid b$ but $a$ doesn't divide $c$, then $a$ doesn't divide $b+c$.
4. Prove that a positive integer $a$ is odd if and only if it can be expressed as a difference of consecutive squares. (Here, consecutive squares means the squares of two positive integers $k, k+1$ ).
5. It is known that integer 1 is not divisible by any positive integer other than 1 . Use it to prove:
(a) If integer $n$ is even, then $n+1$ is not divisible by 2 .
(b) If integer $n$ is divisible by 5 , then $n+1$ is not divisible by 5 .
(c) If $n+1$ is divisible by 5 , then $n$ is not divisible by 5 .
(d) For any integer $n>1, x$ and $n+1$ have no common factors other than 1. (A common factor is a positive integer $k$ that divides both of the integers in question.)
6. Prove or disprove the following statement: $\forall p \in \mathbb{Z}$ ( $p$ is prime $\Longrightarrow p+1$ is not a power of 2)
7. (a) Prove that if $x, y \in \mathbb{R}$ are such that $x y=0$, then $x=0 \vee y=0$. [Hint: $x$ has an inverse.]
(b) Prove (using nothing but the basic facts about reals given above) that $x^{2}-5 x+6=0$ if and only if $x=2$ or $x=3$. Here $x$ is a real number.
8. Recall that the product of any two positive real numbers is positive. Given real numbers $x$, $y, z$, such that $y>z$ and $x y<x z$, prove that $x<0$.
9. The standard card deck has 52 cards ( 4 suits, each with 13 values, from 6 to ace). You have drawn 2 cards, which happened to be king of spades and 9 of hearts.

You now draw 3 more cards. What is the probability that your 5 -card hand will contain the following combinations:
(a) Four of a kind: four cards of the same value
(b) Three of a kind: three card of the same value (but not 4 cards of the same value)
(c) Two kings and two nines (but no three of a kind)
(d) Three kings and two nines.

