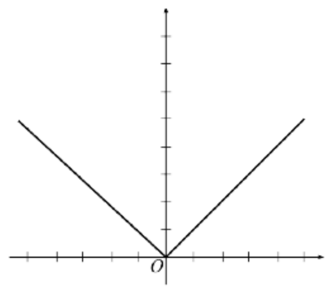
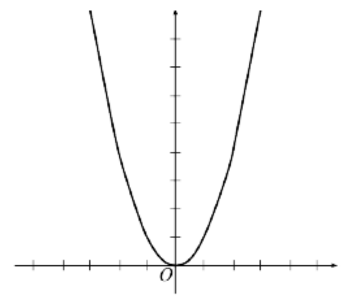
MATH 7
PARABOLA

Graphs of $y = |x|$ and $y = x^2$

The figure below shows graphs of functions $y = |x|$ and $y = x^2$; the latter graph is called a *parabola*.





Transformations of graphs

- Graph of function $y = f(x + a)$ is obtained from graph of $y = f(x)$ by shifting horizontally by a units to the left; for example, graph of $y = (x+1)^2$ is parabola with vertex at $(-1, 0)$.
- Graph of function $y = f(x) + k$ is obtained from graph of $y = f(x)$ by shifting vertically by k units up; for example, graph of $y = x^2 + 1$ is parabola with vertex at $(0, 1)$.
- Finally, graph of $y = kf(x)$ is obtained from graph of $y = f(x)$ by rescaling vertically by factor of k; if k is negative, it means flip upside down and then rescale by factor of $|k|$.

Combining these results, we can sketch the graph of any quadratic function, which will also be a parabola. To sketch it, we need to complete the square, writing

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a} = a(x - h)^2 + k, \qquad h = -\frac{b}{2a}, \quad k = -\frac{b^2 - 4ac}{4a}$$

For example: $x^2 + x = (x + \frac{1}{2})^2 - \frac{1}{4}$

The result will be a parabola obtained by stretching the usual parabola vertically by factor a (if $a < 0$, this means flipping it upside down and then stretching by $|a|$) and then moving it so that the vertex will be at point (h, k),

In particular, the branches go up if $a > 0$ and down if $a < 0$.

Homework

1. For what values of a does the polynomial $x^2+ax+14$ has no roots? exactly one root? two roots?

2. Sketch the graphs of the folowing functions and relations:

(a) $x+y=4$	(b) $\|x\|+y=4$	(c) $x^2+4x+y^2-4y=0$
(d) $y=\|x-5\|$	(e) $y=\|x+1\|+\|x-1\|$	(f) $y=x^2-x$
(g) $y=\|x^2-x\|$	(h) $y=x^2-5x+6$	(i) $y=-2x^2+8x+6$

3. Solve the following equations and inequalities

(a) $x^2-x+6\geq 0$ (b) $\dfrac{2x+1}{x-5}\leq 0$ (c) $x^4-3x^2+8=0$

(d) $x(x-2)(x+18)>0$

4. Find all intersection points of parabola $y=x^2$ and the circle with radius $\sqrt{6}$ and center at $(0,4)$.

5. Prove that for any point P on the parabola $y=\frac{x^2}{4}+1$, the distance from P to the x-axis is equal to the distance from P to the point $(0,2)$.